



# Universality and evolution of Sivers effect

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Brookhaven National Laboratory

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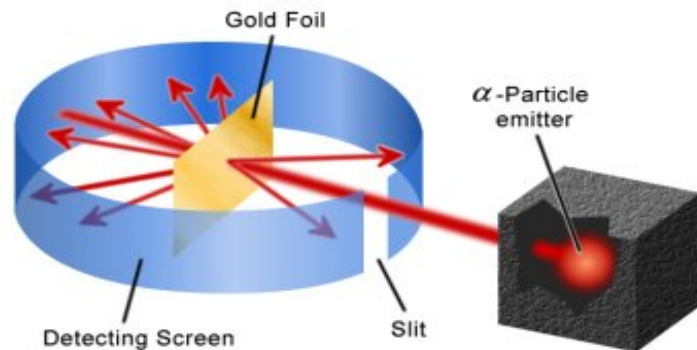
# Outline

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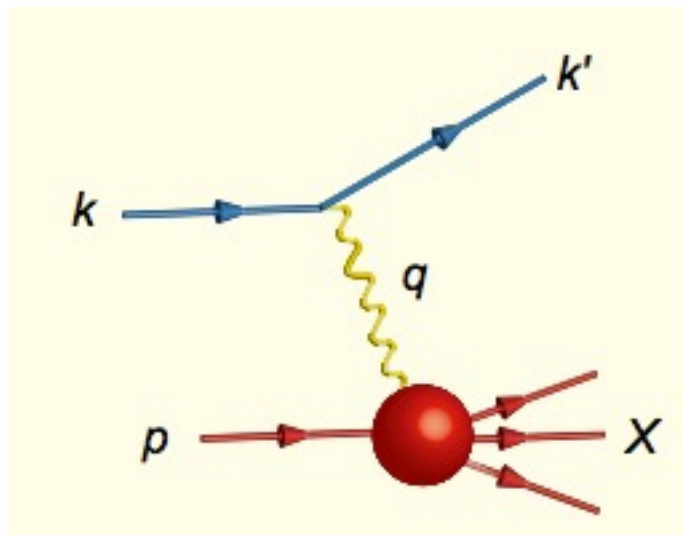
- Introduction
- Sivers function and sign change
- QCD evolution of TMDs: status and progress
- Summary

# High energy scattering: one way to study the structure of matter

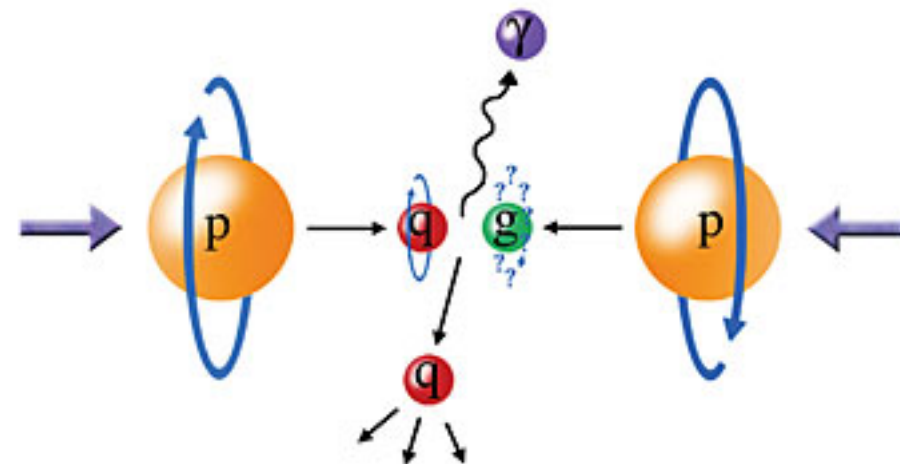
- Originated from Rutherford's experiment (1911)
  - Atomic structure: atomic nucleus (proton and neutron - nucleon)



- To extract information on the nucleon structure, we send in a probe and measure the outcome of the collisions



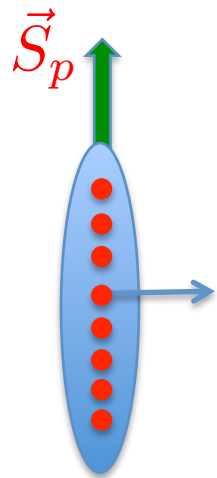
Deep Inelastic Scattering (DIS)



Proton-Proton collisions

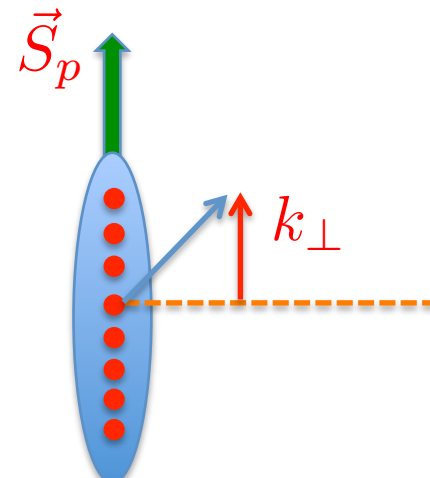
# New trend on hadron structure

- Hadron structure: one-dimensional picture to three-dimensional tomography



$$f(x)$$

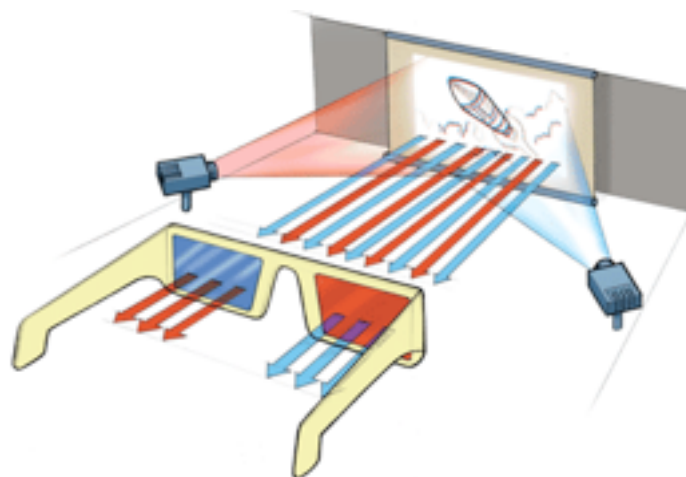
Collinear PDFs



$$f(x, k_{\perp})$$

Transverse momentum distributions (TMDs)

- A seemingly simple extension, very interesting and non-trivial consequences: much richer QCD dynamics and hadron structure



## Universality property

- Collinear PDFs are universal (process-independent): they are the same in all processes, such as SIDIS, DY, and pp collisions
- Some TMDs can be different in different processes
  - Sivers function: unpolarized parton distribution in transversely polarized proton
  - Boer-Mulders function: transversely polarized parton distribution in unpolarized proton
  - Naive time-reversal odd TMDs, change sign between SIDIS and DY

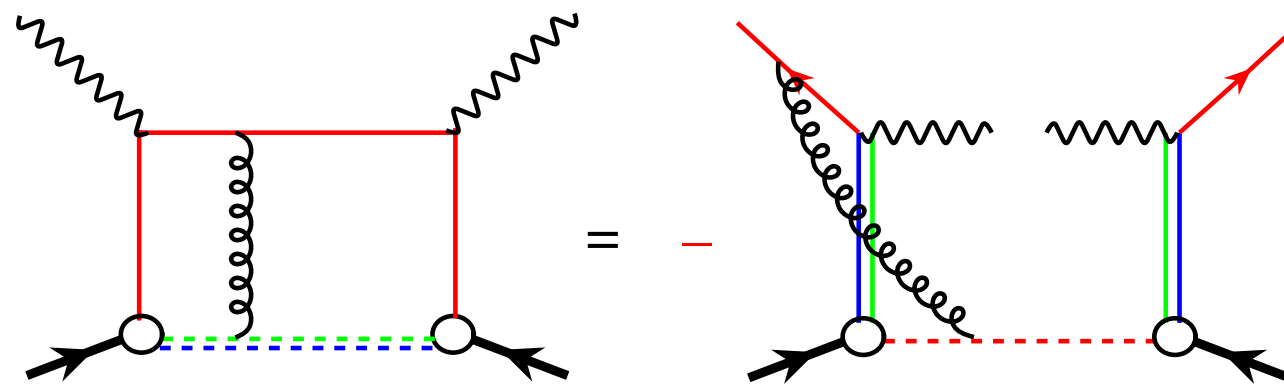
		quark pol.		
		U	L	T
nucleon pol.	U	$f_1$		$h_1^\perp$
	L		$g_{1L}$	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	$g_{1T}$	$h_1, h_{1T}^\perp$

# Sivers function

- Sivers function: an asymmetric parton distribution in a transversely polarized nucleon (kt correlated with the spin of the nucleon)

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/h^\uparrow}(x, k_\perp) \vec{S} \cdot \hat{p} \times \hat{\mathbf{k}}_\perp$$

**Spin-independent**
**Spin-dependent**

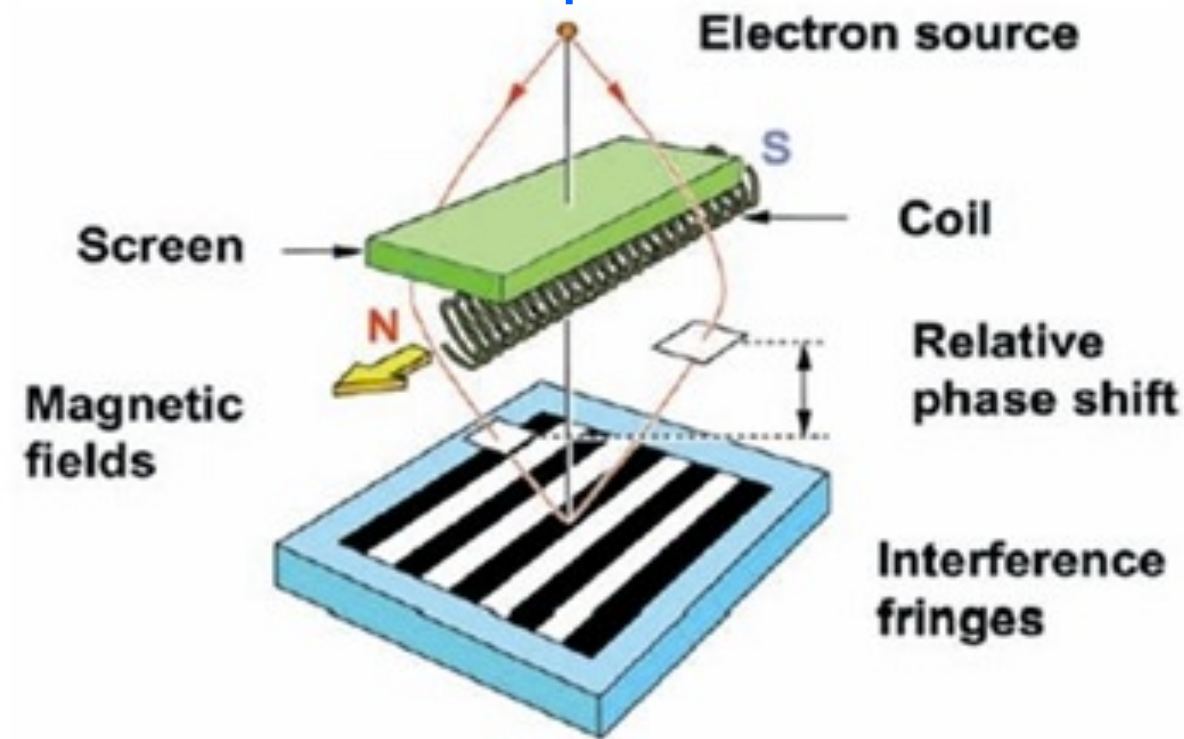


$$\Delta^N f_{q/h^\uparrow}^{\text{DIS}}(x, k_\perp) = - \Delta^N f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp)$$

NSAC milestone: most important property of the Sivers function, need to be tested

# Sivers effect is a QCD version of Aharonov-Bohm effect

- Pure quantum effect: different paths lead to interference



- Physics Today, September 2009



feature  
article

# The Aharonov–Bohm effects: Variations on a subtle theme

Herman Batelaan and Akira Tonomura

The notion, introduced 50 years ago, that electrons could be affected by electromagnetic potentials without coming in contact with actual force fields was received with a skepticism that has spawned a flourishing of experimental tests and expansions of the original idea.

Aharonov stresses that the arguments that led to the prediction of the various electromagnetic AB effects apply equally well to any other gauge-invariant quantum theory. In the standard model of particle physics, the strong and weak nuclear interactions are also described by gauge-invariant theories. So one may expect that particle-physics experimenters will be looking for new AB effects in new domains.

*We thank Adam Caprez for the artwork. This article is based on work supported by the NSF under grant no. 0653182.*





## Test two things

$$\Delta^N f_{q/h^\uparrow}^{\text{DIS}}(x, k_\perp) = \textcircled{-} \Delta^N f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp)$$

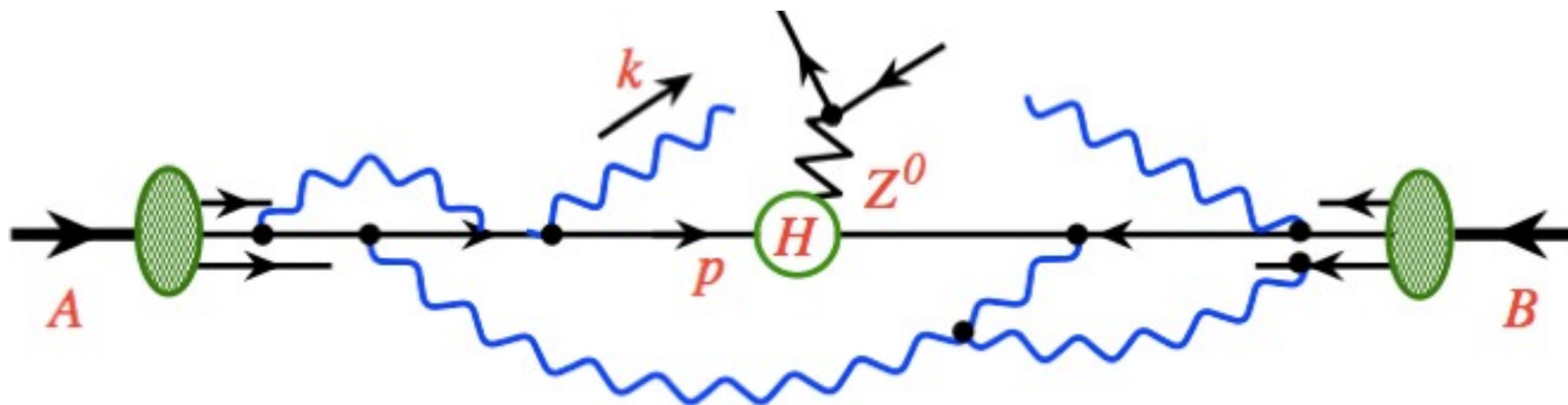
- Sign change
- The magnitude is the same: QCD evolution is important

$$F(x, k_\perp; Q)$$

TMD is probed at a specific scale  $Q$   
at different  $Q$ , they are different

# Why QCD evolution is needed

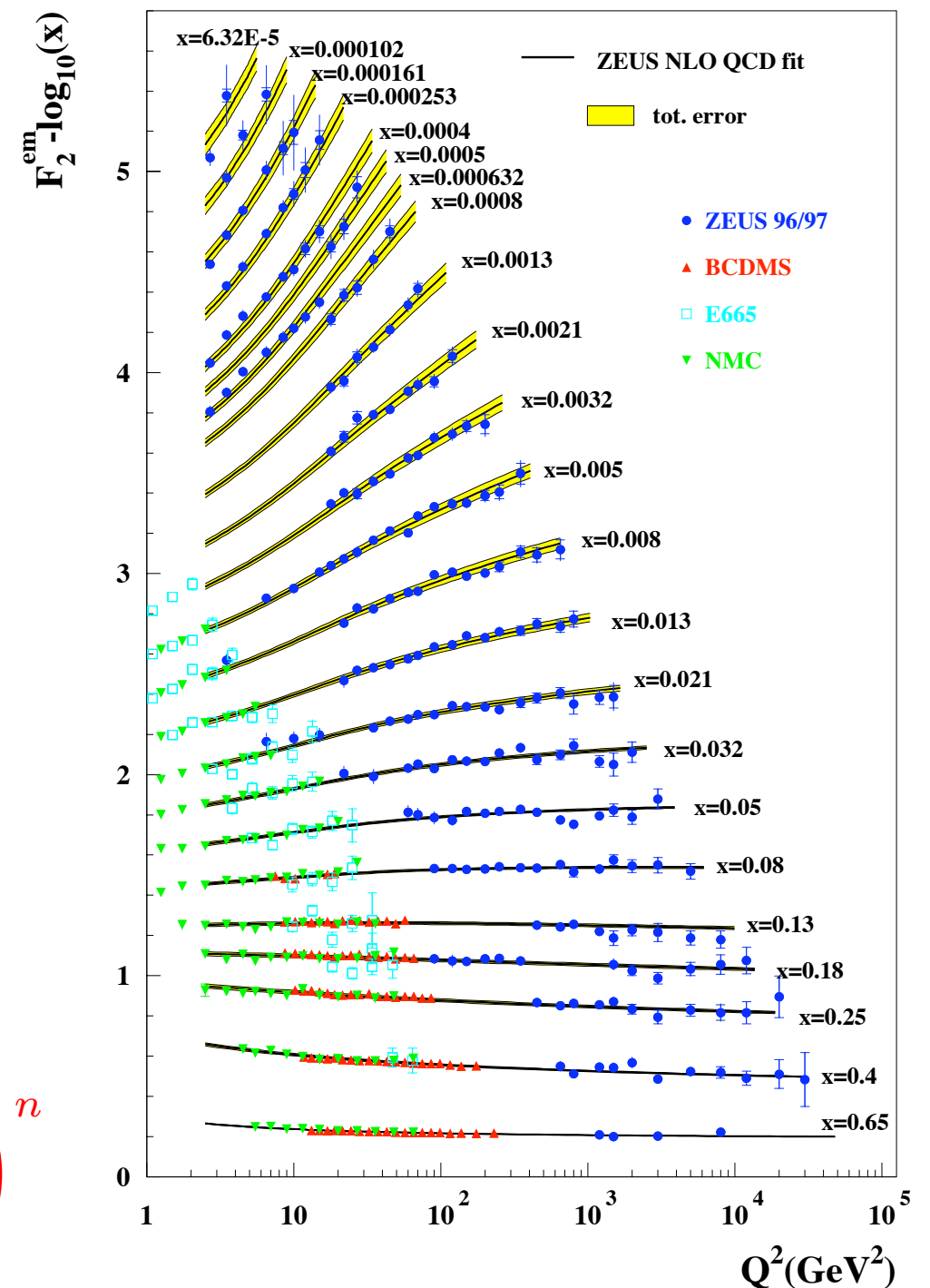
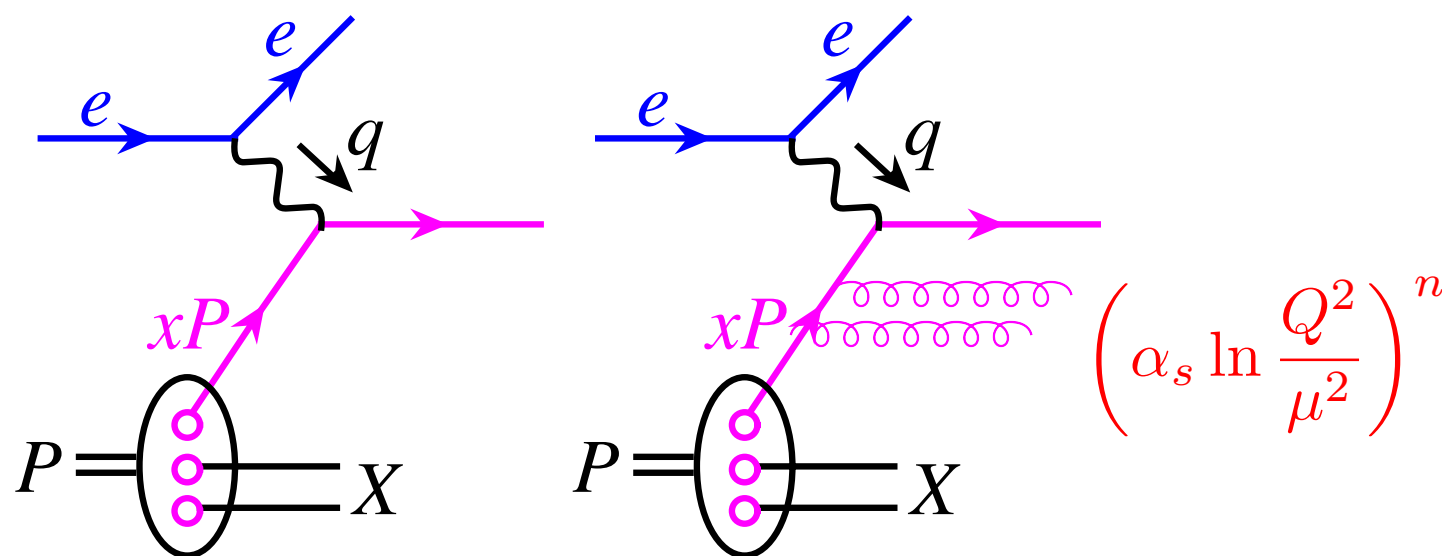
- Experiments are operated in different energy and kinematic regions, to make reliable predictions, one has to take into account these differences
  - $Q$  is different:  $Q \sim 1 - 3$  GeV in SIDIS,  $Q \sim 10$  GeV at  $e^+e^-$ ,  $Q \sim 4 - 90$  GeV for DY, W/Z
  - Also  $\sqrt{s}$  dependence is important Qiu-Zhang 1999, ResBos
- We use the energy evolution equation for the relevant parton distribution functions (PDFs) or fragmentation function (FFs) to account for the kinematic differences



# QCD evolution: meaning

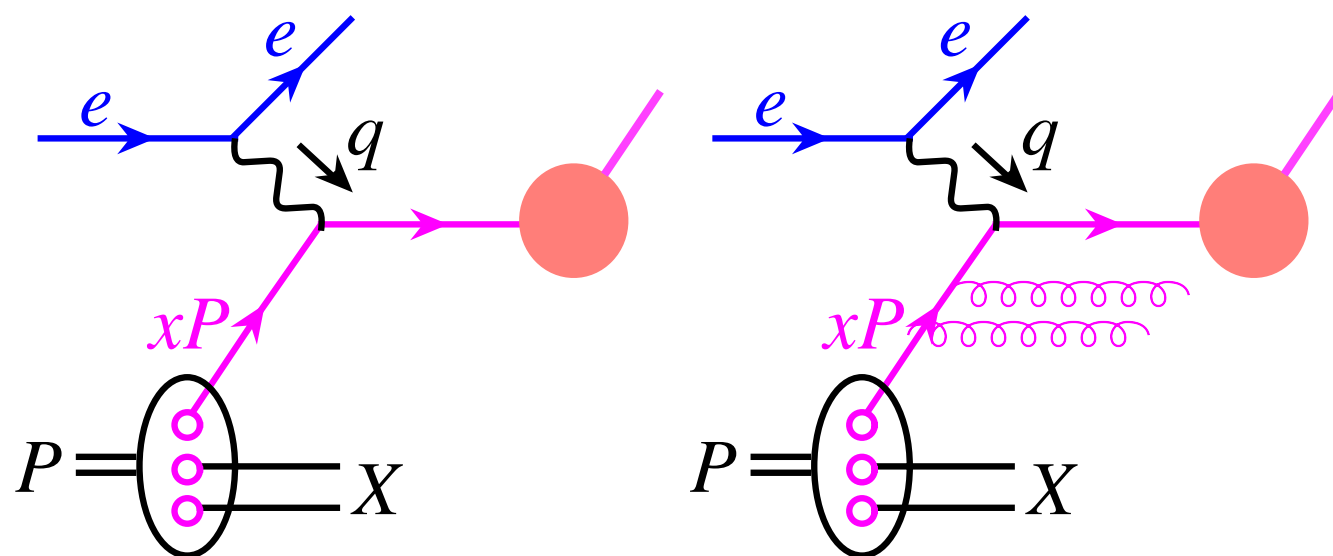
## ■ What is QCD evolution of TMDs anyway?

- Evolution = include important perturbative corrections
- One of the well-known examples is the DGLAP evolution of collinear PDFs, which lead to the scaling violation observed in inclusive DIS process
- What it does is to resum the so-called single logarithms in the higher order perturbative calculations



# QCD evolution: TMDs

- TMD factorization works in the situation where there are two observed momenta in the process, such as SIDIS, DY, W/Z production and in the kinematic region where  $Q \gg q_T$
- Evolution again = include important perturbative corrections
- What it does is to resum the so-called double logarithms in the higher order perturbative corrections
- For SIDIS:  $q_T$  is the transverse momentum of the final-state hadron



$$\left( \alpha_s \ln^2 \frac{Q^2}{q_T^2} \right)^n$$



# Many approaches for TMD evolution

- Collins-Soper-Sterman (CSS) resummation framework

Collins-Soper-Sterman 1985

ResBos: C.P. Yuan, P. Nadolsky

Qiu-Zhang 1999, Vogelsang ...

Kang-Xiao-Yuan 2011, Sun-Yuan 2013

- New Collins approach

Aybat-Rogers 2011,

Aybat-Collins-Rogers-Qiu, 2012

Aybat-Prokudin-Rogers 2012

- Soft Collinear Effective Theory (SCET)

Echevarria-Idilbi-Schafer-Scimemi 2012

They are all consistent with each other perturbatively  
However, they could have very different phenomenological predictions

# What the evolution looks like?

- We have a TMD distribution  $F(x, k_{\perp}; Q)$  measured at a scale  $Q$

- It is easy to deal in the Fourier transformed space

$$F(x, b; Q) = \int d^2 k_{\perp} e^{-i k_{\perp} \cdot b} F(x, k_{\perp}; Q)$$

- Standard CSS formalism tells us it evolves from an initial scale  $\mu_b = c/b$   
 $c = 2e^{-\gamma_E} \sim O(1)$

$$F(x, b; Q) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\}$$

$$A = \sum_{n=1} A^{(n)} \left( \frac{\alpha_s}{\pi} \right)^n, \quad B = \sum_{n=1} B^{(n)} \left( \frac{\alpha_s}{\pi} \right)^n$$

$$A^{(1)} = C_F$$

$$A^{(2)} = \frac{C_F}{2} \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R n_f \right]$$


$$B^{(1)} = -\frac{3}{2} C_F$$

# Connection to other approaches: new Collins

- Derive new Collins evolution from CSS

$$F(x, b; Q_f) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\}$$

$$F(x, b; Q_i) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^{Q_i} \frac{d\mu}{\mu} \left( A \ln \frac{Q_i^2}{\mu^2} + B \right) \right\}$$


$$F(x, b; Q_f) = F(x, b; Q_i) \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left( \frac{Q_f^2}{Q_i^2} \right)^{- \int_{c/b}^{Q_i} \frac{d\mu}{\mu} A}$$

- This is the same as in SCET



## Connection to other approaches: Sun-Yuan

- To derive Sun-Yuan evolution kernel, choose lowest order  $A^{(1)}$ ,  $B^{(1)}$

$$I(b, Q_i, Q_f) = \left( \frac{Q_f^2}{Q_i^2} \right)^{\int_{c/b}^{Q_i} \frac{d\mu}{\mu} C_F \frac{\alpha_s(\mu)}{\pi}} = \text{Exp} \left[ \ln \left( \frac{Q_f^2}{Q_i^2} \right) \int_{c/b}^{Q_i} \frac{d\mu}{\mu} C_F \frac{\alpha_s(\mu)}{\pi} \right]$$

$$\ln \left( \frac{Q_f^2}{Q_i^2} \right) = 2 \int_{Q_i}^{Q_f} \frac{d\mu}{\mu}$$

Ignore the scale-dependence in coupling constant

$$\int_{c/b}^{Q_i} \frac{d\mu}{\mu} C_F \frac{\alpha_s(\mu)}{\pi} = C_F \frac{\alpha_s}{\pi} \int_{c/b}^{Q_i} \frac{d\mu}{\mu} = \frac{1}{2} C_F \frac{\alpha_s}{\pi} \ln \left( \frac{Q_i^2 b^2}{c^2} \right)$$

Now you “might” put the scale back into coupling constant

$$I(b, Q_i, Q_f) = \text{Exp} \left[ C_F \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \frac{\alpha_s}{\pi} \ln \left( \frac{Q_i^2 b^2}{c^2} \right) \right]$$

- In general, Sun-Yuan could be different from new Collins evolution
  - perturbatively at  $O(\alpha_s)$  they are exactly the same
  - should work fine for not-too-wide- $Q$  range

see also, Aidala-Field-Gamberg-Rogers, arXiv:1401.2654

# What's the complication in QCD evolution?

- So far the evolution kernel is calculated in perturbation theory, so valid only for small  $b$  region:

$$F(x, b; Q_f) = F(x, b; Q_i) \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left( \frac{Q_f^2}{Q_i^2} \right)^{- \int_{c/b}^{Q_i} \frac{d\mu}{\mu} A}$$

- Fourier transform back to the momentum space, one needs the whole  $b$  region (also large  $b$ ): need some non-perturbative extrapolation

$$\begin{aligned} F(x, k_\perp; Q) &= \frac{1}{(2\pi)^2} \int d^2 b e^{i k_\perp \cdot b} F(x, b; Q) \\ &= \frac{1}{2\pi} \int_0^\infty db b J_0(k_\perp b) F(x, b; Q) \end{aligned}$$

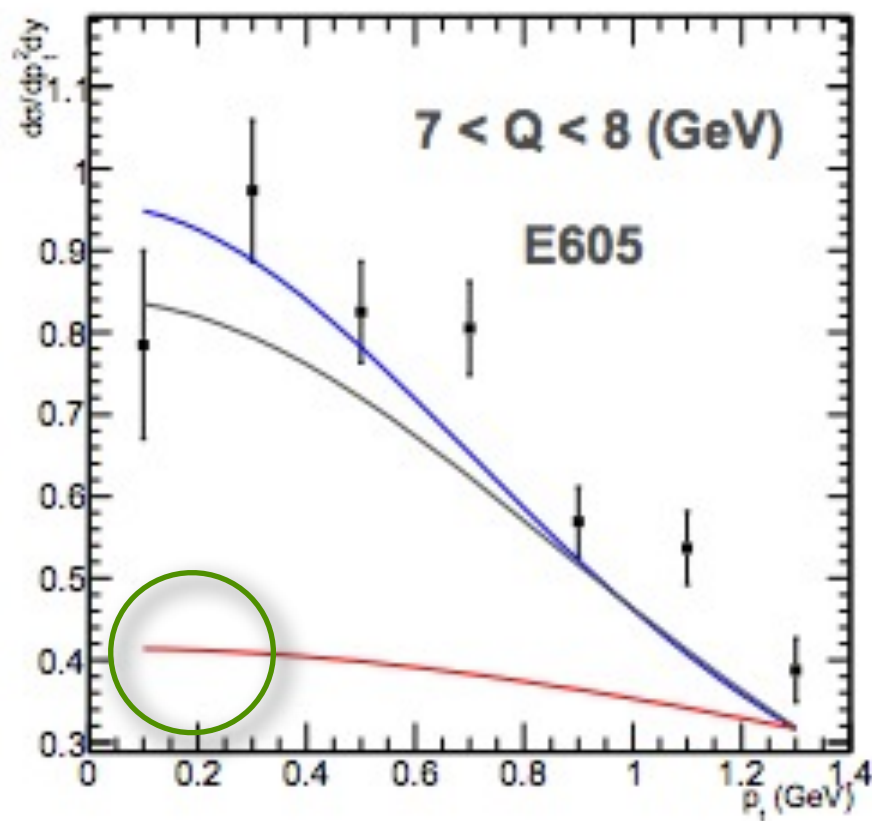
- Widely used prescription (CSS):

$$F(x, b; Q_f) = F(x, b; Q_i) R^{\text{pert}}(b_*, Q_i, Q_f) \times R^{NP}(b, Q_i, Q_f)$$

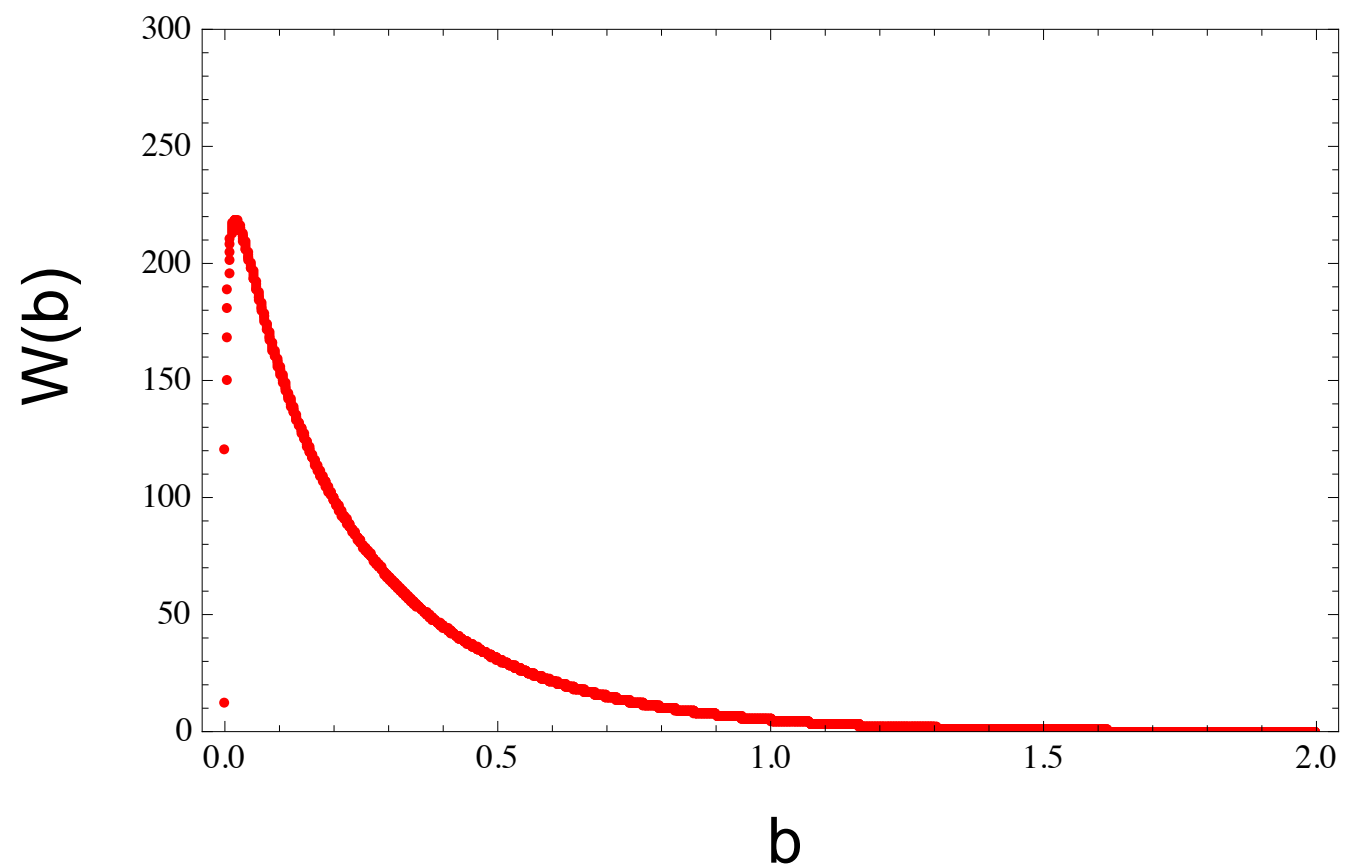
$$b_* = b / \sqrt{1 + (b/b_{\text{max}})^2}$$

# New Collins evolution + Gaussian ansatz

- Choose some Gaussian form for TMDs at initial scale  $Q_0$ , then evolve to  $W/Z$  scale, to see if it describes the  $p_t$  distribution
  - It does not (use a reasonable  $b_{\max}$ ). It always leads to a rather flat  $p_t$  distribution: the integrand in  $b$ -space is almost a delta-function concentrated at  $b=0$
  - It will then lead to a rather flat  $p_t$  distribution: curvature much smaller than data



See, e.g., [arXiv:1308.5003](https://arxiv.org/abs/1308.5003)



## Use conventional CSS formalism

- In the conventional CSS formalism, one further calculate TMD at  $c/b$  scale in terms of collinear PDFs

$$F(x, b; Q) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\}$$

- At LO, we have  $F(x, b; c/b) = F(x, \mu = c/b)$
- Choose non-perturbative Sudakov function

$$F(x, b; Q) = F(x, c/b_*) R^{\text{pert}}(Q, b_*) R^{\text{NP}}(Q, b)$$

$$R^{\text{NP}}(Q, b) = \exp(-S^{\text{NP}})$$

- Typical simplest form for unpolarized PDF and FF

$$S_{pdf}^{\text{NP}} = b^2 \left[ g_1^{pdf} + \frac{g_2}{2} \ln(Q/Q_0) \right]$$

$$S_{ff}^{\text{NP}} = b^2 \left[ g_1^{ff}/z^2 + \frac{g_2}{2} \ln(Q/Q_0) \right]$$

# Intuitive meaning of these parameters

- Let us understand these parameters

$$S_{pdf}^{NP} = b^2 \left[ g_1^{pdf} + \frac{g_2}{2} \ln(Q/Q_0) \right] \quad S_{ff}^{NP} = b^2 \left[ g_1^{ff} / z^2 + \frac{g_2}{2} \ln(Q/Q_0) \right]$$

$g_1^{pdf} = \langle k_{\perp}^2 \rangle / 4$  intrinsic transverse momentum width for PDFs at scale  $Q_0$

$g_1^{ff} = \langle p_{\perp}^2 \rangle / 4$  intrinsic transverse momentum width for FFs at scale  $Q_0$

$g_2$  mimic the increase in the width observed by the experiments  
large  $Q$  leads to more shower

- Sivers asymmetry is very sensitive to  $g_2$  (though the Drell-Yan unpolarized cross section is not)
  - Choose a wrong  $g_2$  leads to very different result



## Tune the parameters to describe all data

- Now we will try to tune these parameters to describe all the world data for pt distribution for SIDIS, DY, W/Z at all energies
  - Let us choose  $Q_0^2 = 2.4 \text{ GeV}^2$ , i.e., the HERMES scale
  - At this scale, the intrinsic transverse momentum width is already extracted by different group: there are some freedom

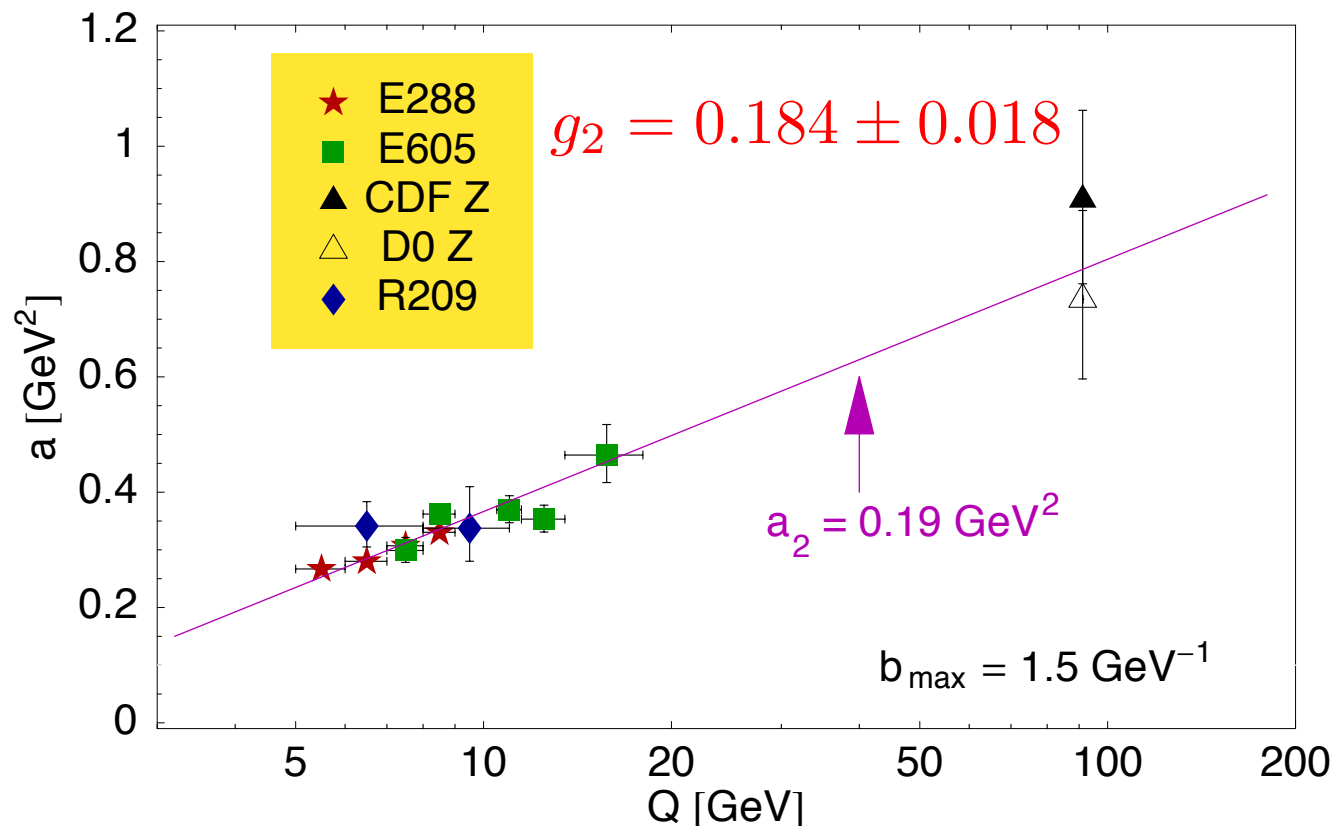
arXiv:1003.2190 & Torino

$$\langle k_{\perp}^2 \rangle = 0.25 - 0.44 \text{ GeV}^2$$

$$\langle p_T^2 \rangle = 0.15 - 0.2 \text{ GeV}^2$$

# Finding a way to describe both SIDIS and DY/WZ

- Study unpolarized cross section, and pin-down  $g_2$ 
  - Slightly adjust  $g_2$  (within their fitted uncertainty) such that non-perturbative Sudakov can predict  $\langle k_{\perp}^2 \rangle$  at HERMES
  - Once this is fixed, adjust  $\langle p_T^2 \rangle$  such that it gives a good description of SIDIS



hep-ph/0506225

$$g_2 \approx 0.16$$

$$g_1^{\text{pdf}} = \langle k_{\perp}^2 \rangle / 4$$

$$g_1^{\text{ff}} = \langle p_{\perp}^2 \rangle / 4$$

$$\langle k_{\perp}^2 \rangle = 0.38 \text{ GeV}^2$$

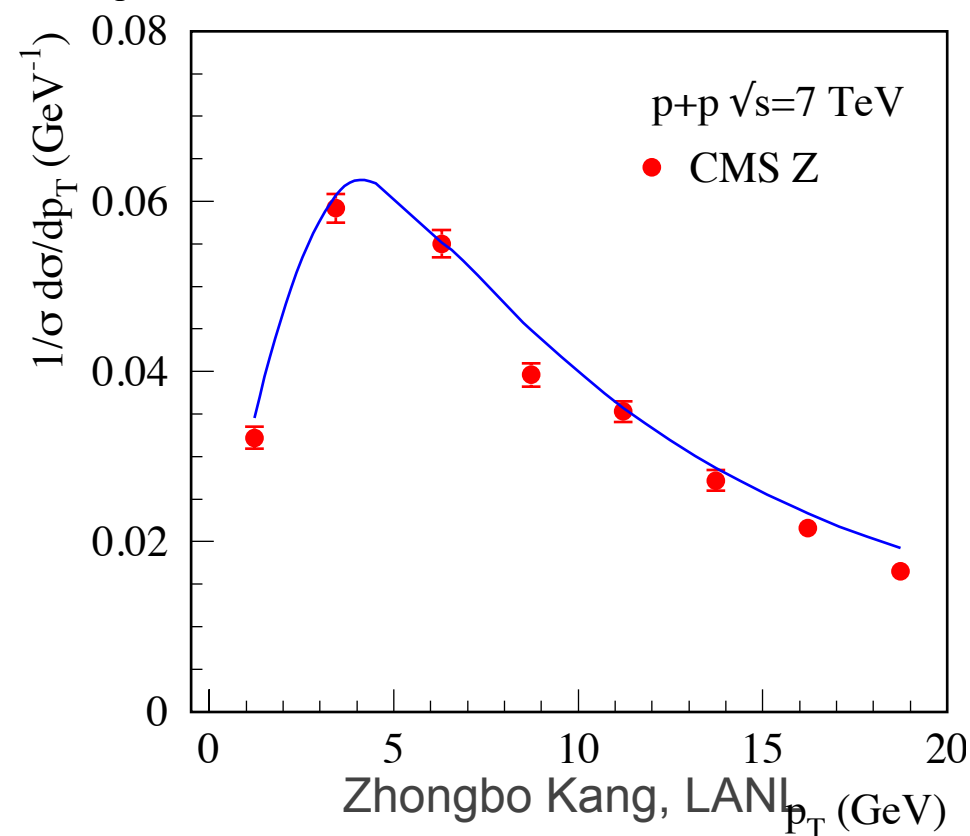
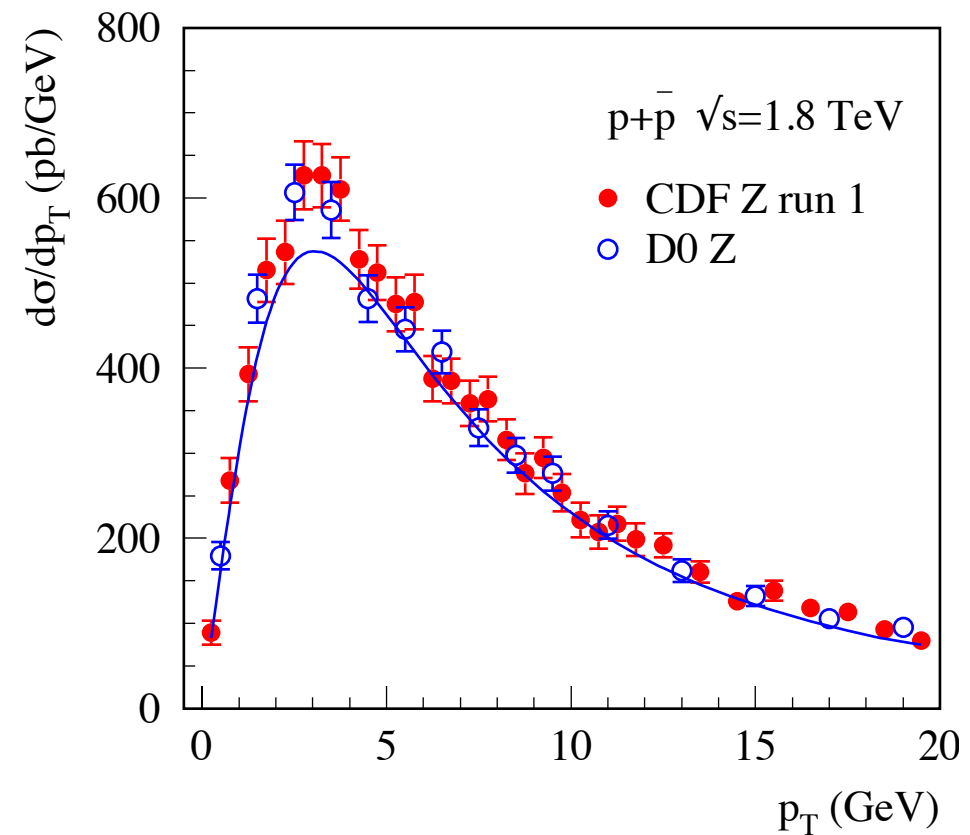
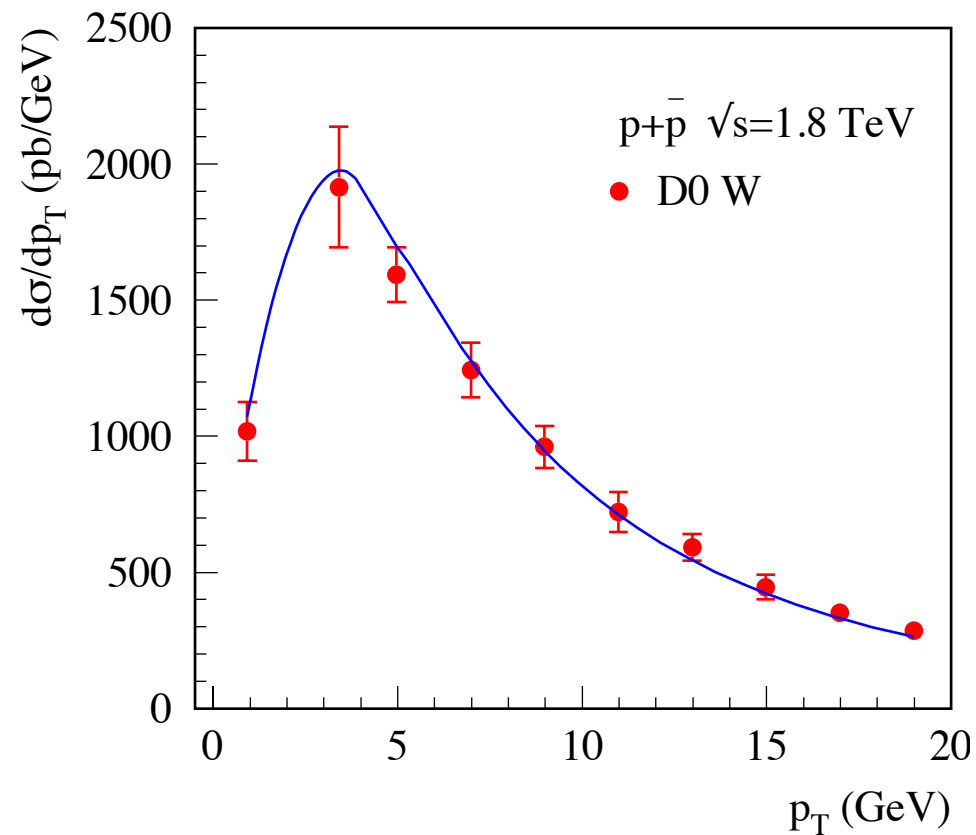
$$\langle p_{\perp}^2 \rangle = 0.19 \text{ GeV}^2$$

consistent with  
arXiv:1309.3507

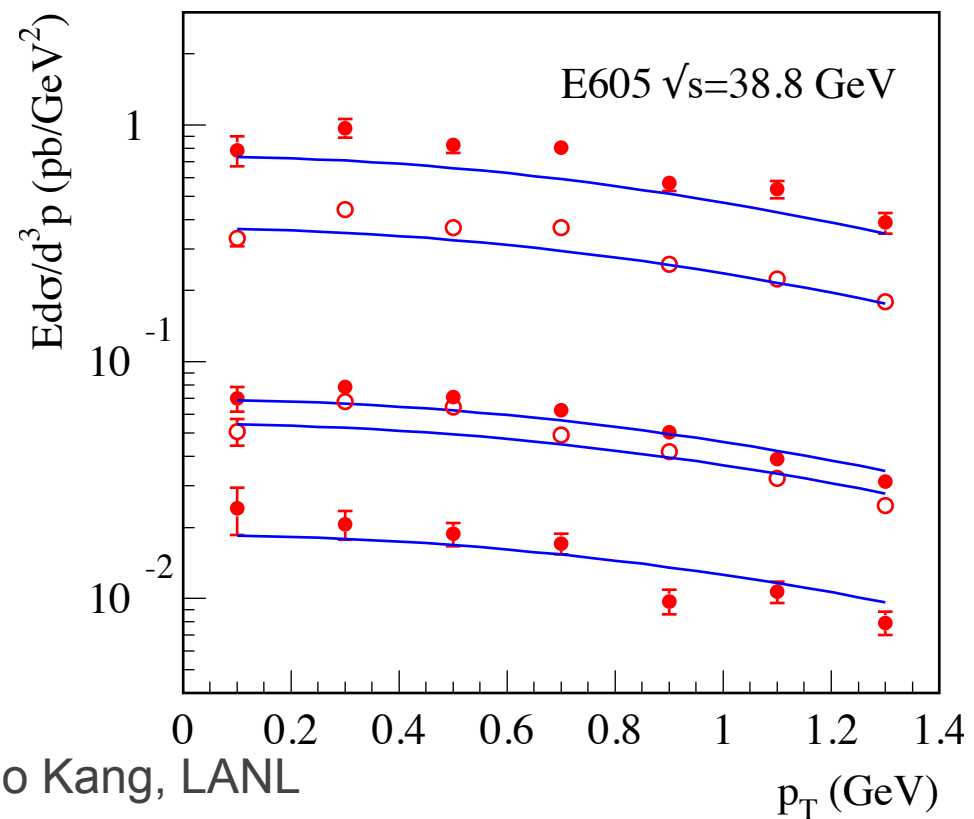
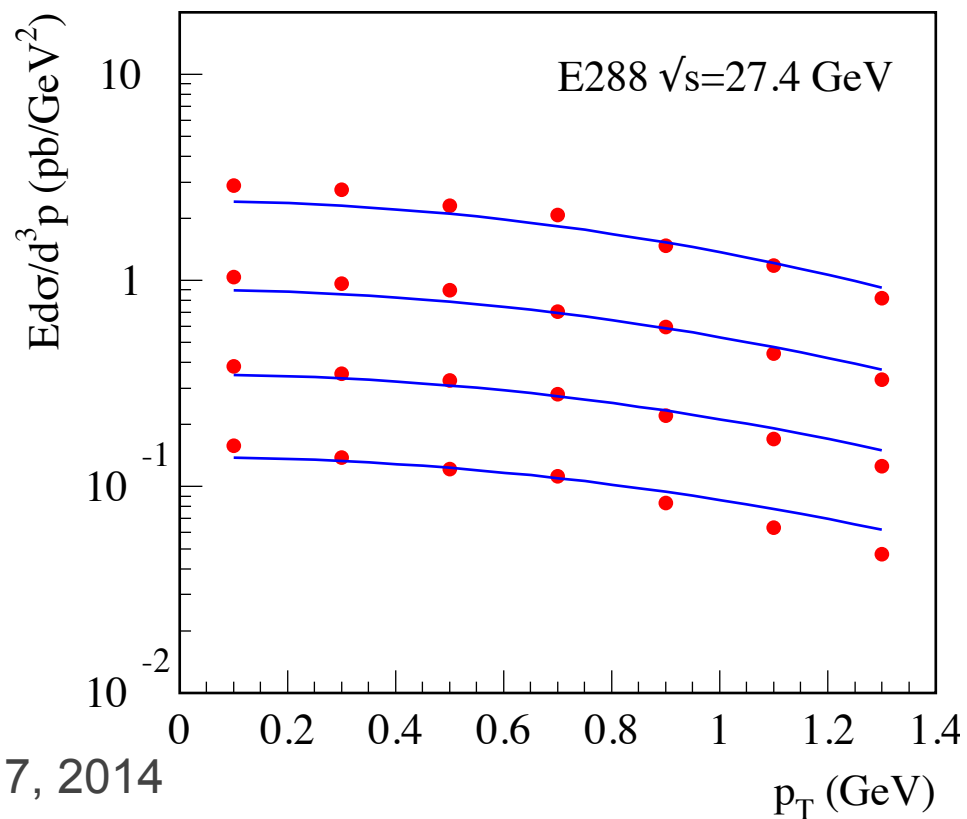
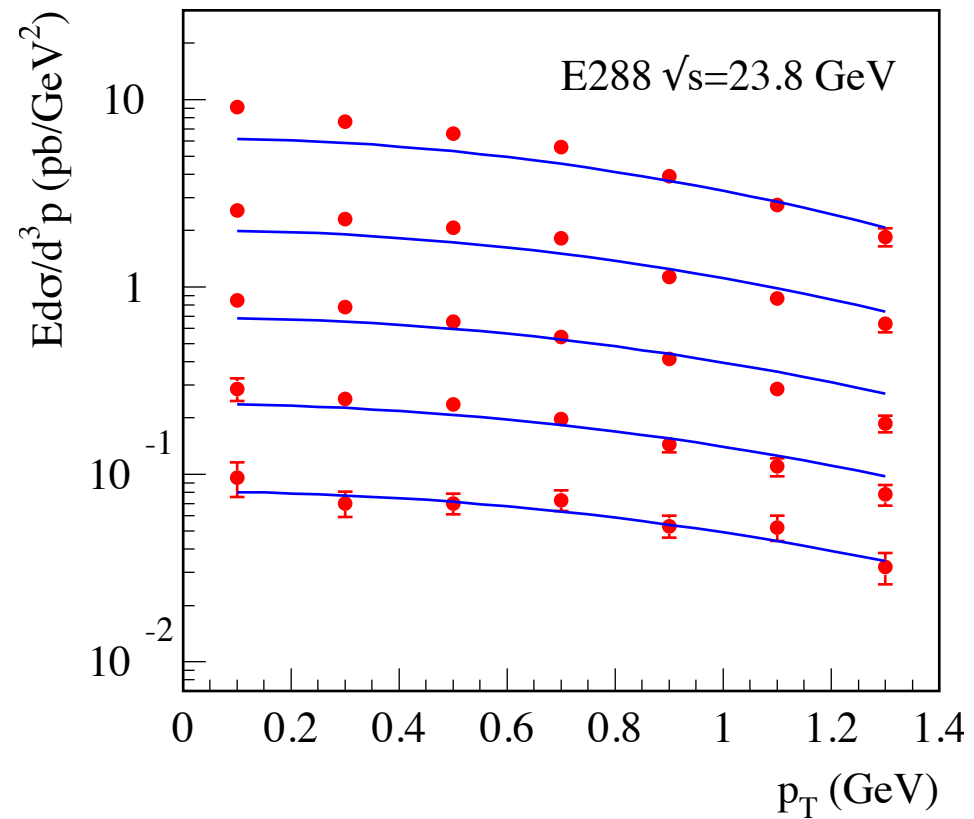
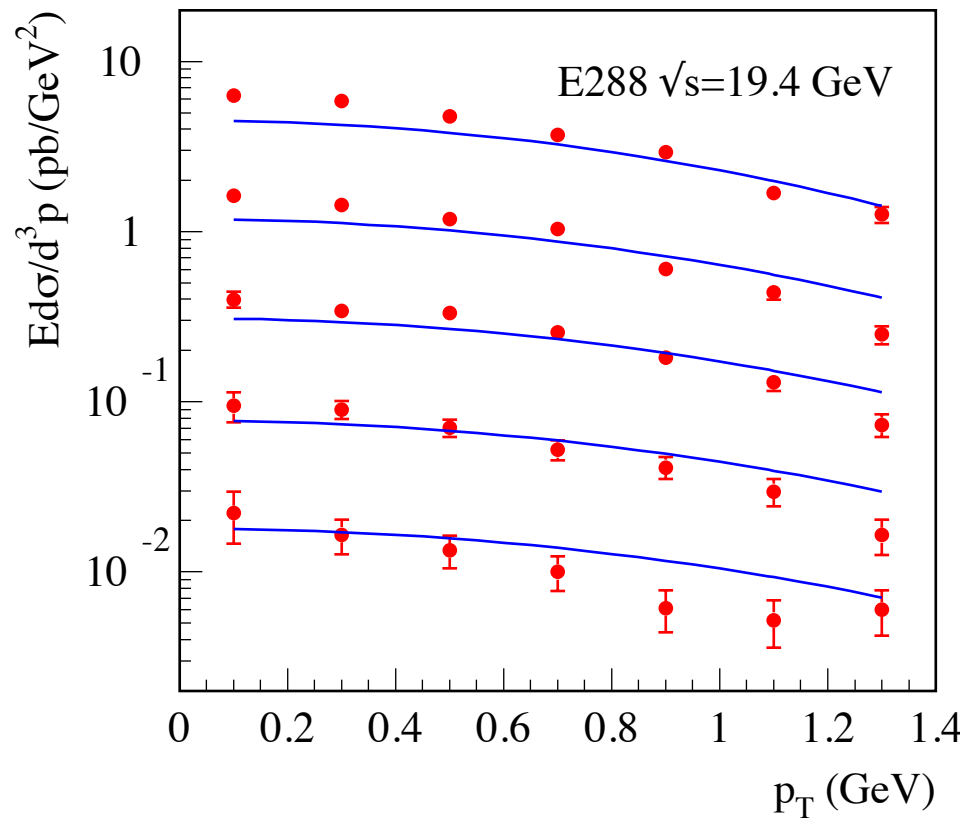


This actually works

■ Description of W/Z data at Tevatron and LHC Echevarria-Idilbi-Kang-Vitev, 1401.5078



# Drell-Yan lepton pair production



# Multiplicity distribution in SIDIS 1

## Comparison with COMPASS data

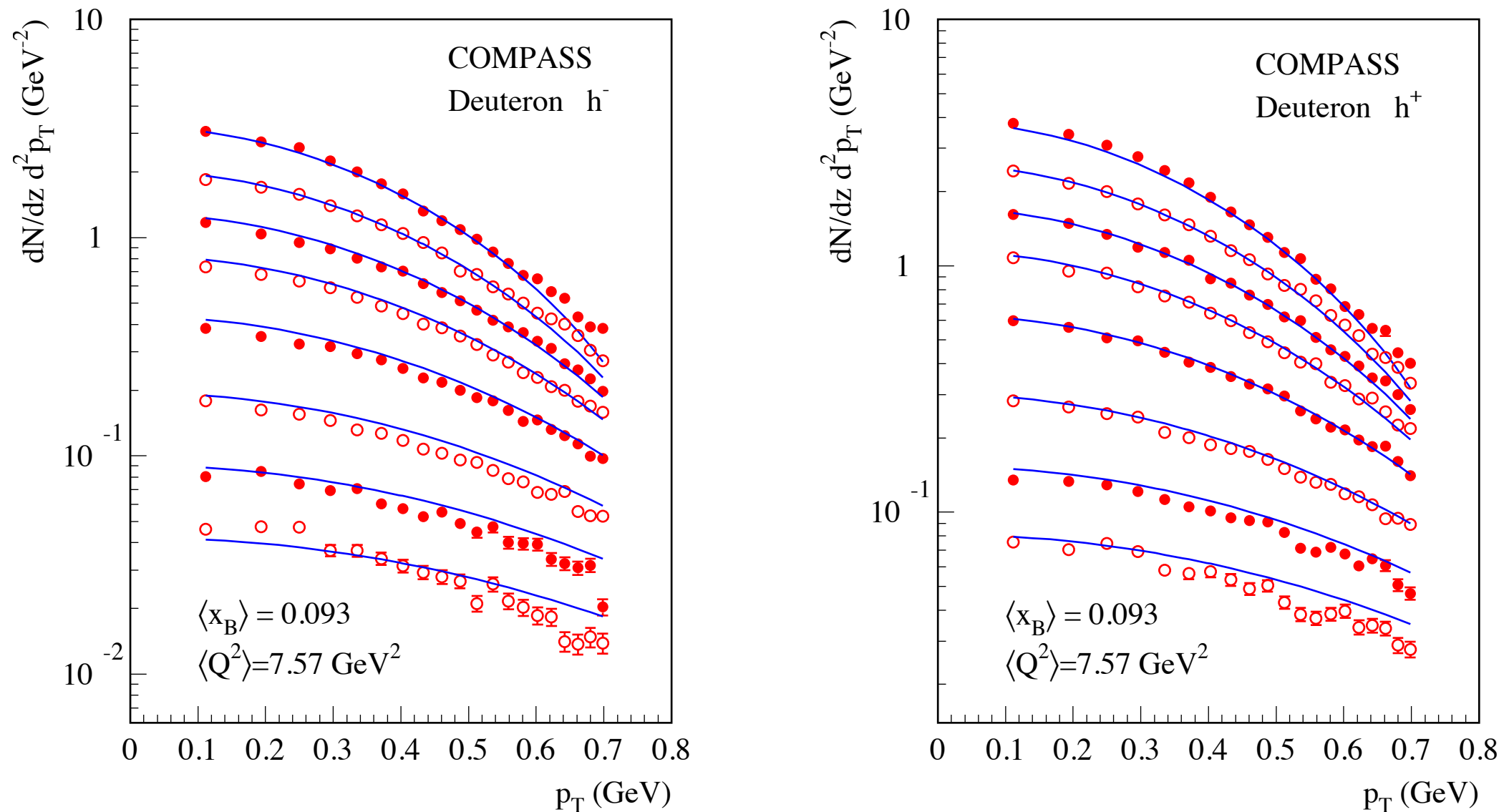


FIG. 2. The comparison with the COMPASS data (deuteron target) [7] at  $\langle Q^2 \rangle = 7.57 \text{ GeV}^2$  and  $\langle x_B \rangle = 0.093$ . The data points from top to bottom correspond to different  $z$  region:  $[0.2, 0.25]$ ,  $[0.25, 0.3]$ ,  $[0.3, 0.35]$ ,  $[0.35, 0.4]$ ,  $[0.4, 0.5]$ ,  $[0.5, 0.6]$ ,  $[0.6, 0.7]$ , and  $[0.7, 0.8]$ .

# Multiplicity distribution in SIDIS 2

## ■ Comparison with HERMES data

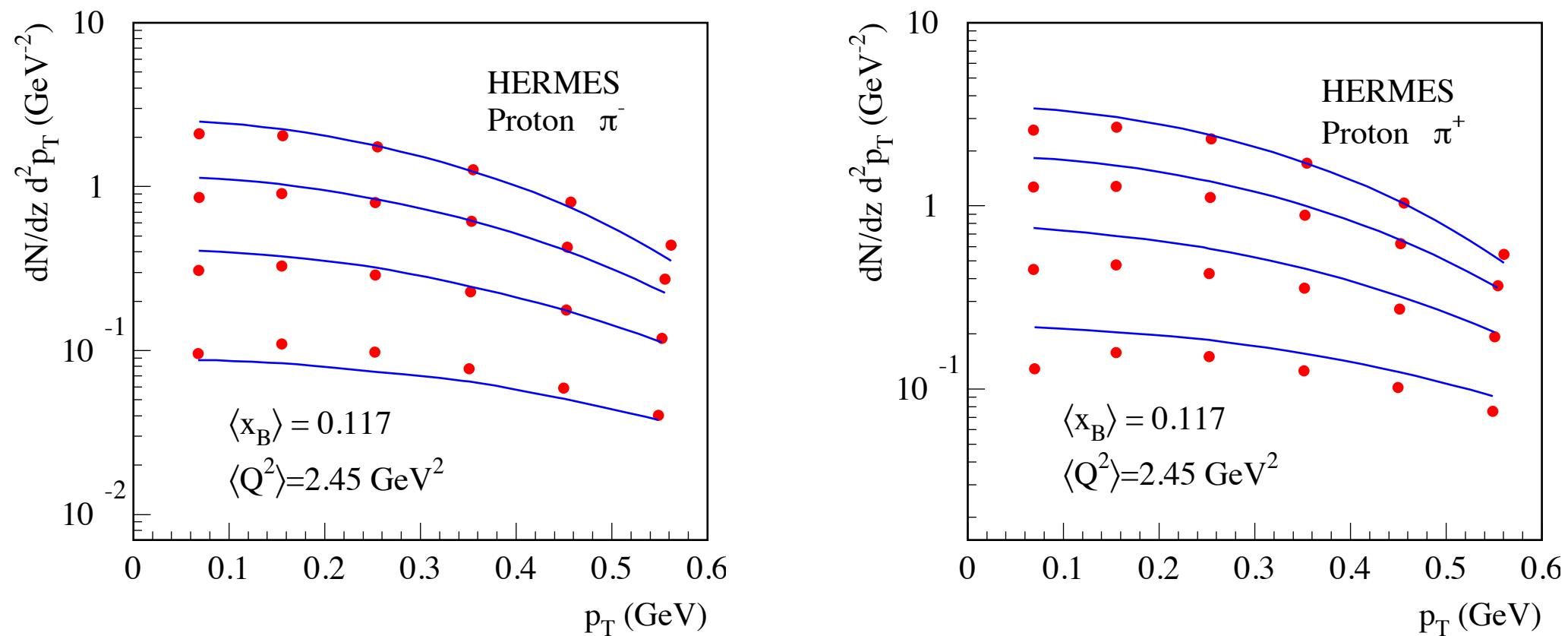


FIG. 1. The comparison with the HERMES data (proton target) [6]. The data points from top to bottom correspond to different  $z$  region:  $[0.2, 0.3]$ ,  $[0.3, 0.4]$ ,  $[0.4, 0.6]$ , and  $[0.6, 0.8]$ .



## Sivers effect

- Now let us try to use the same formalism to describe Sivers effect

$$F(x, b; Q) = F(x, b; c/b) \exp \left\{ - \int_{c/b}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\}$$

- Now  $F(x, b; Q)$  is given by

$$f_{1T}^{\perp q(\alpha)}(x, b; Q) = \frac{1}{M} \int d^2 k_{\perp} e^{-i k_{\perp} \cdot b} k_{\perp}^{\alpha} f_{1T}^{\perp q}(x, k_{\perp}^2; Q)$$

- The perturbative expansion gives Qiu-Sterman function

$$f_{1T}^{\perp q(\alpha)}(x, b; \mu = c/b) = \left( \frac{-i b^{\alpha}}{2} \right) T_{q,F}(x, x, \mu = c/b)$$

# Fitting parameters

- Similar form for non-perturbative Sudakov factor (note:  $g_2$  is spin-independent, so use the same  $g_2$ )

$$S_{\text{NP}}^{\text{sivers}}(b, Q) = b^2 \left[ g_1^{\text{sivers}} + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right] \quad g_1^{\text{sivers}} = \frac{\langle k_{s\perp}^2 \rangle}{4}$$

- Intrinsic  $k_t$ -width for Sivers has to be fitted
  - $x$ -dependence has to be fitted
- Qiu-Sterman function

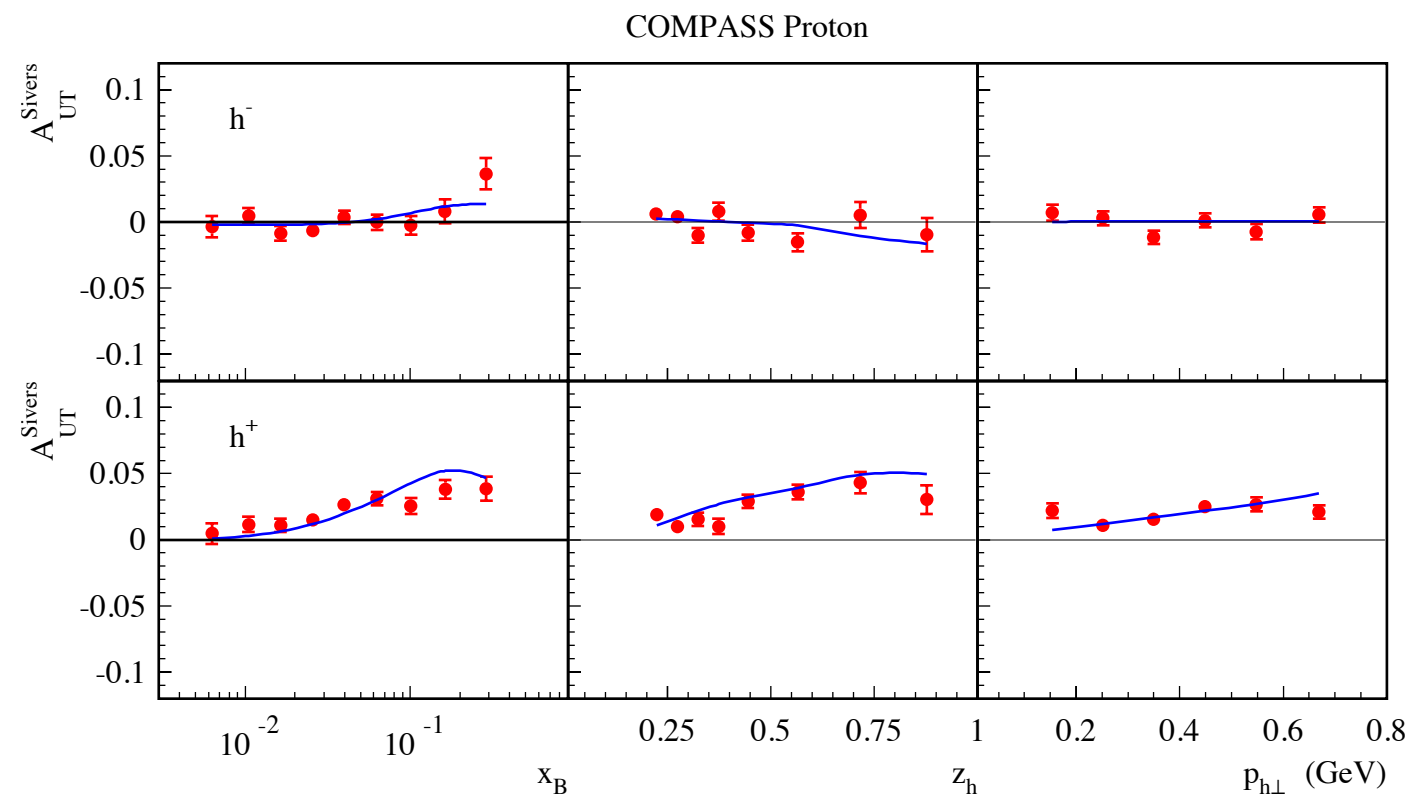
$$T_{q,F}(x, x, \mu) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q} (1 - x)^{\beta_q} f_{q/A}(x, \mu)$$

- Total parameters (11):

$$\langle k_{s\perp}^2 \rangle, N_u, N_d, N_{\bar{u}}, N_{\bar{d}}, N_s, N_{\bar{s}}, \alpha_u, \alpha_d, \alpha_{\text{sea}}, \beta$$

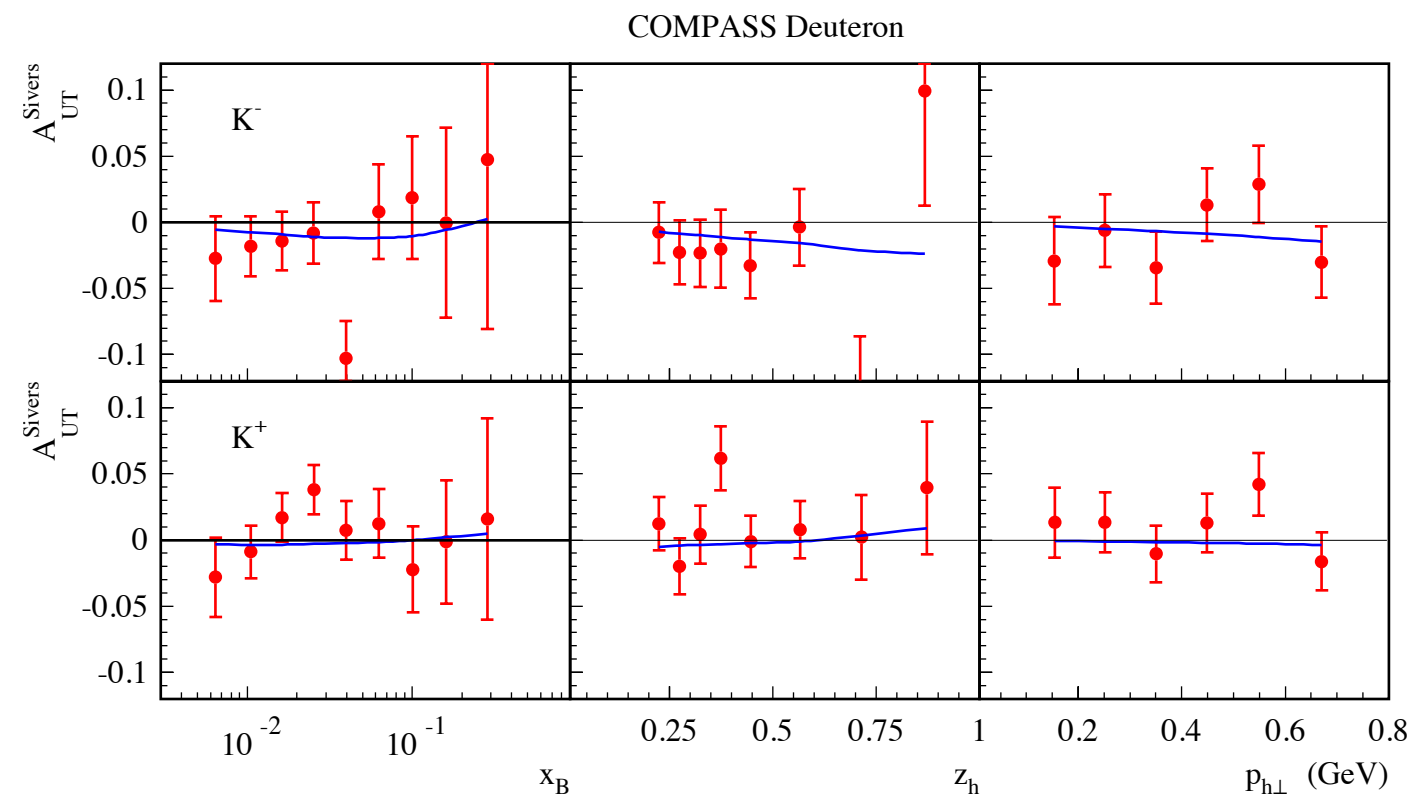
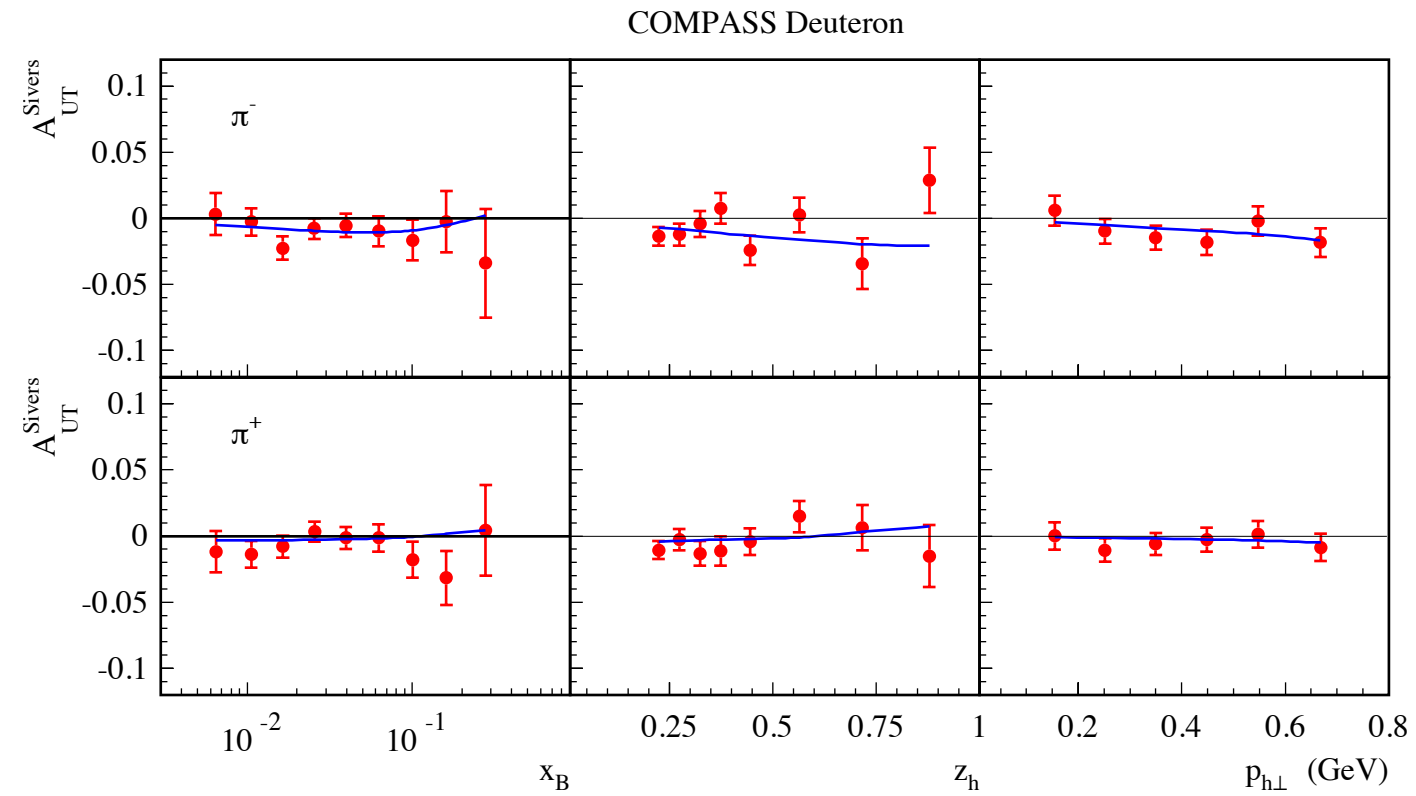
# Fitted results

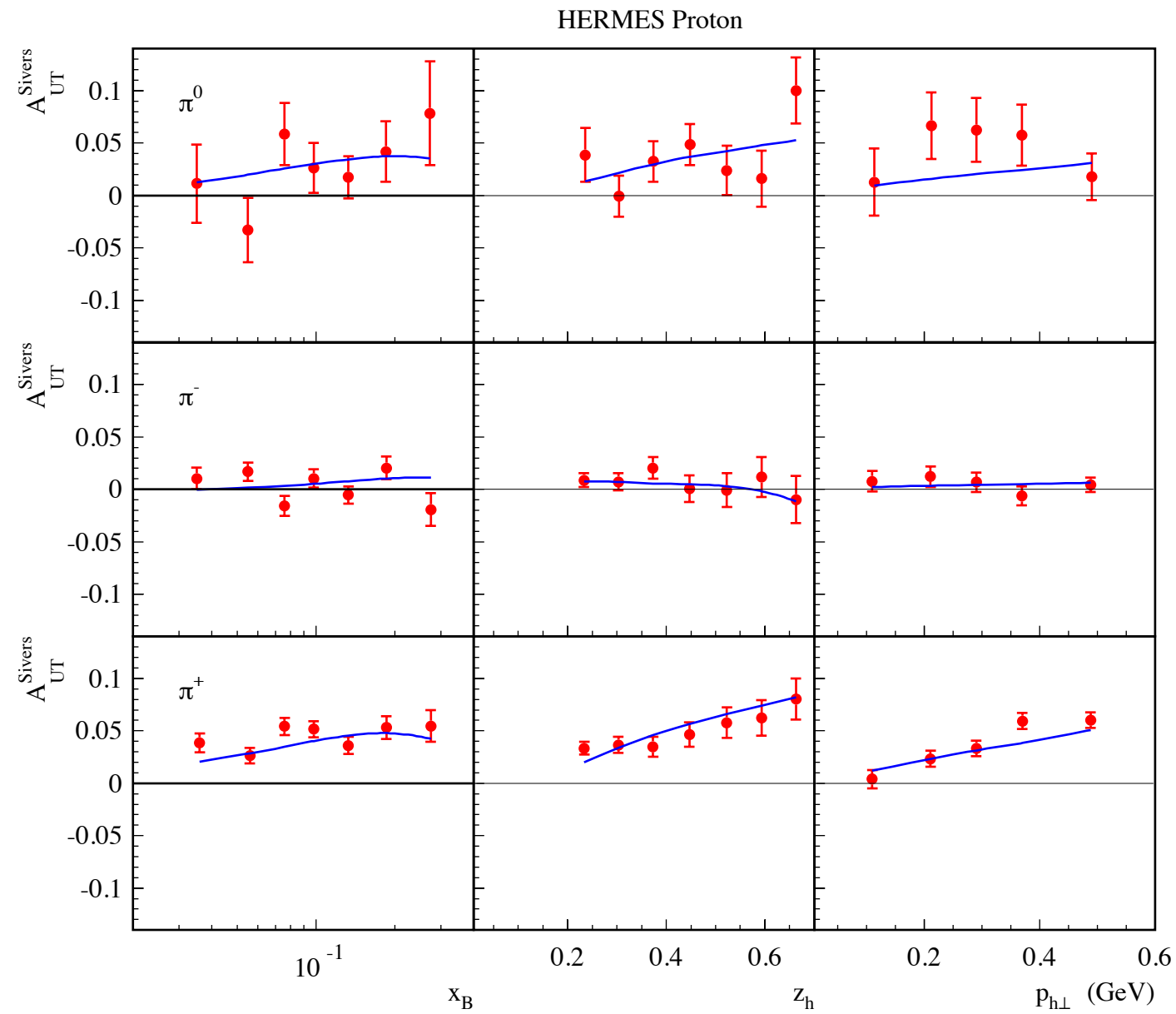
## COMPASS proton



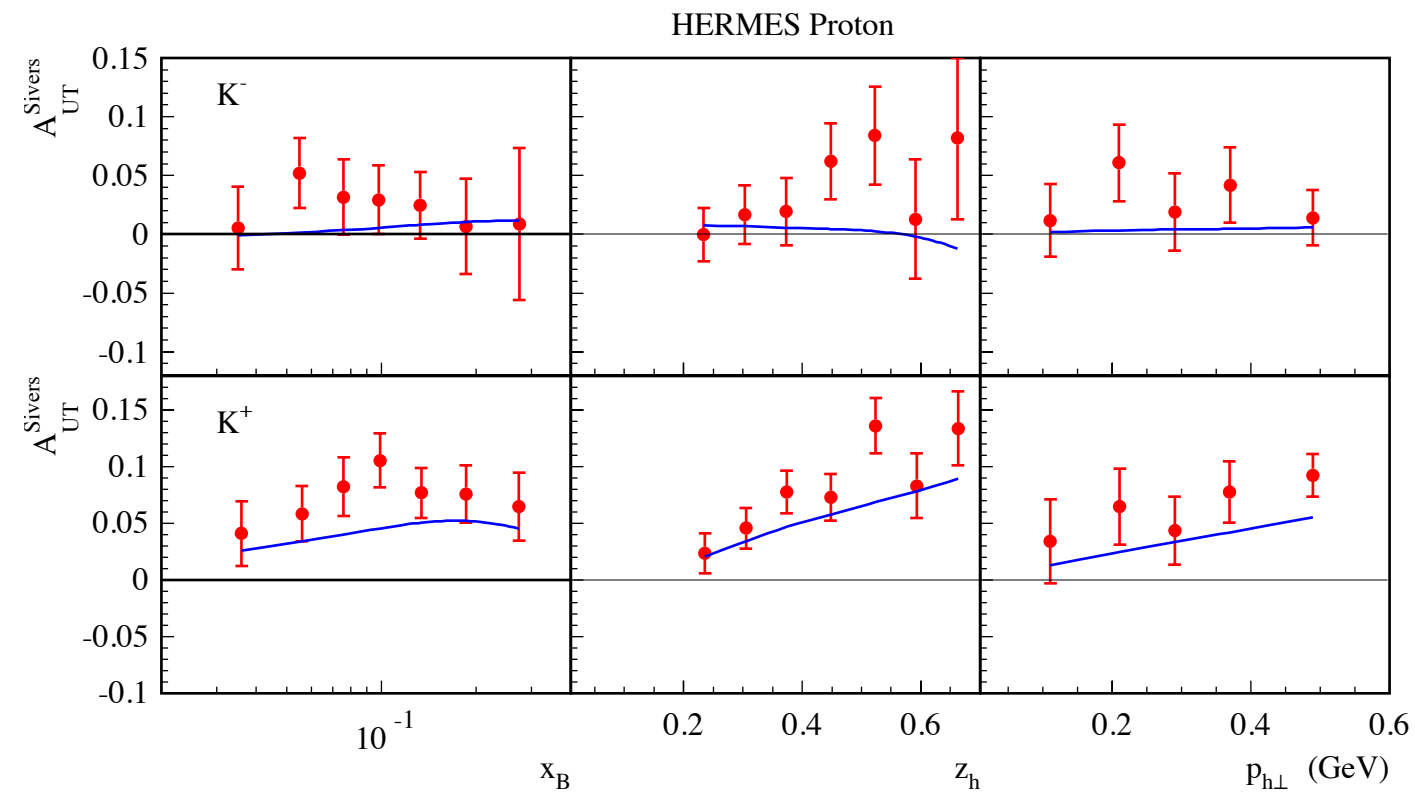


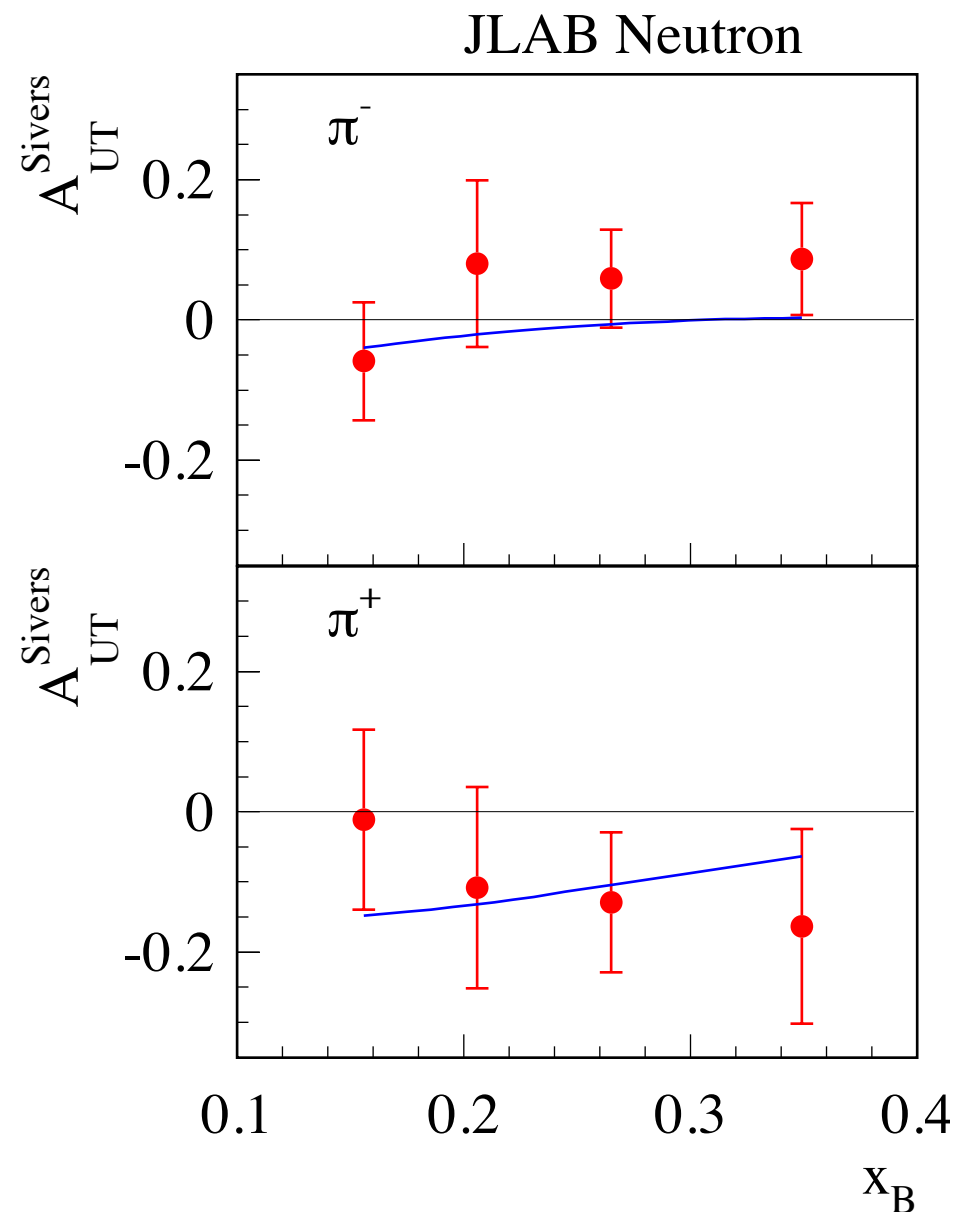
# COMPASS Deuteron target





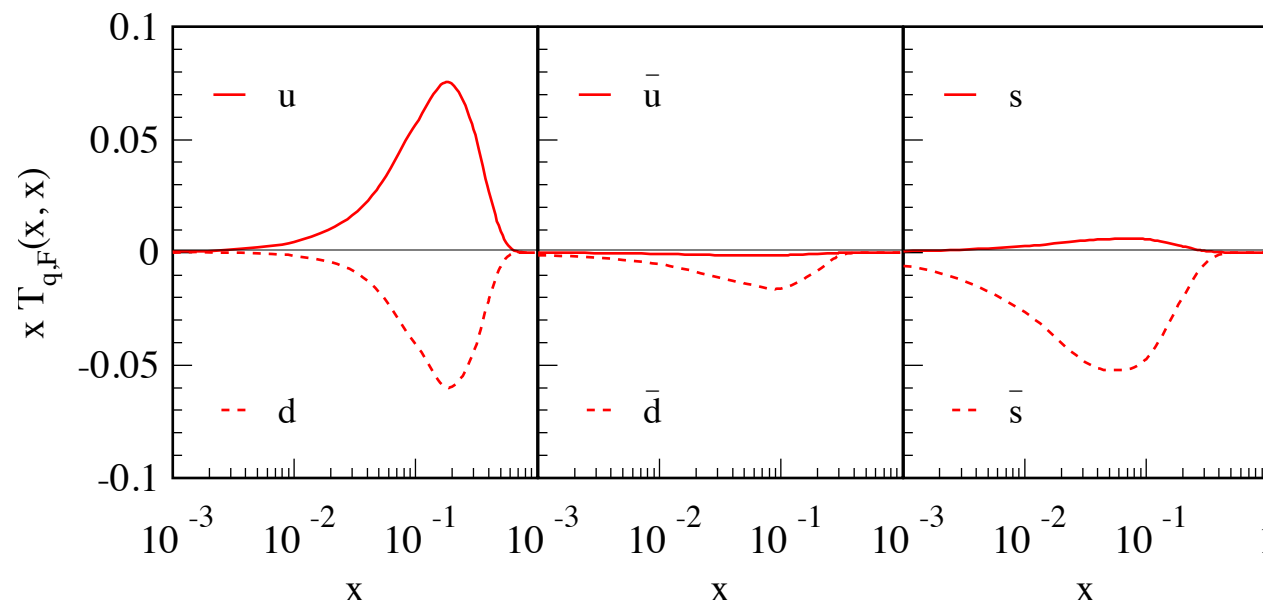
# HERMES Kaons





## Fitted Qiu-Sterman function

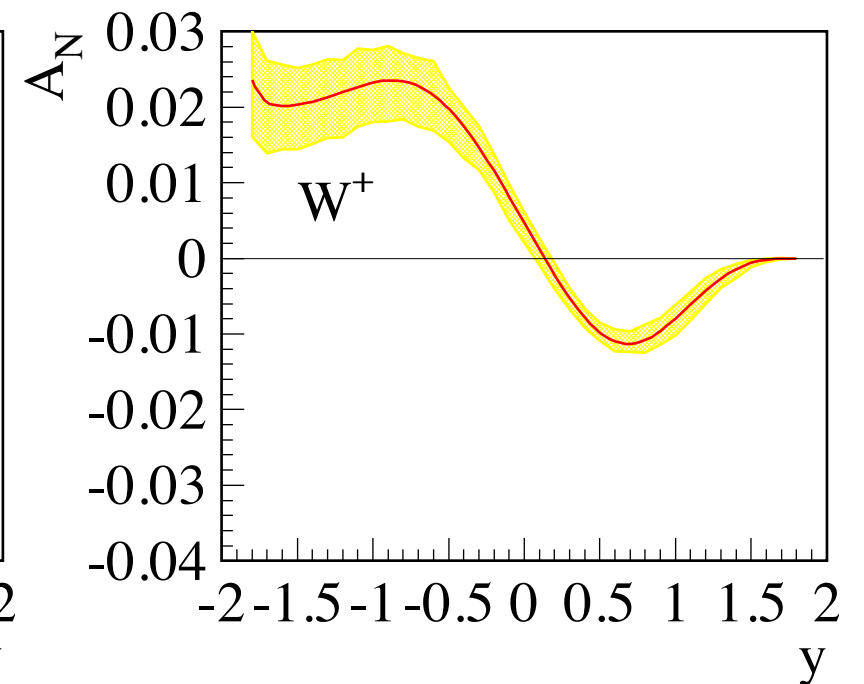
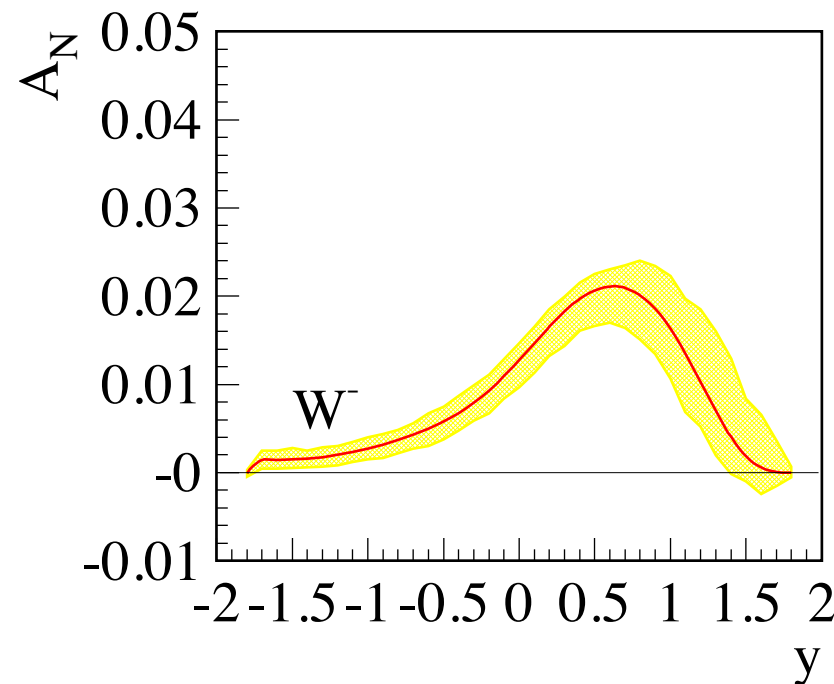
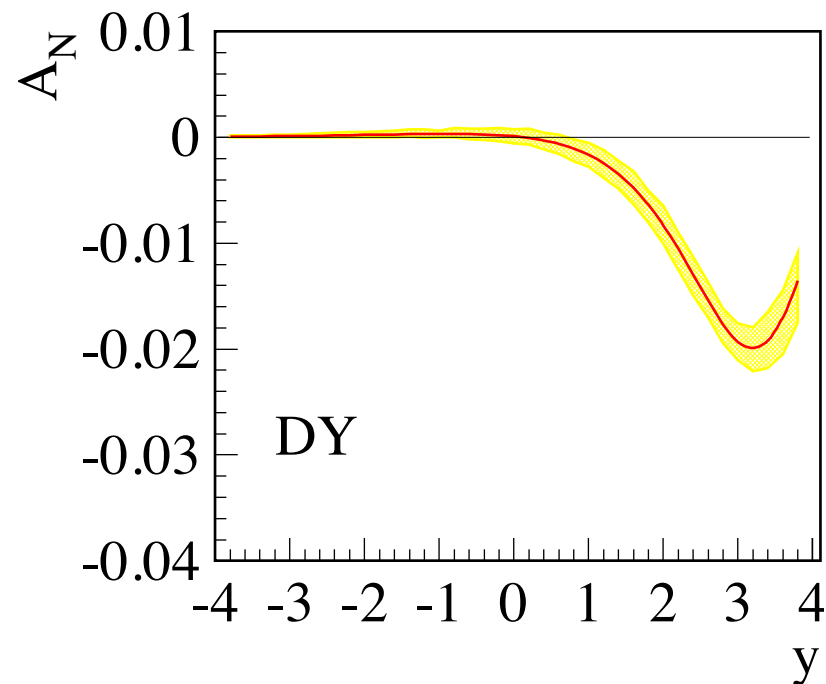
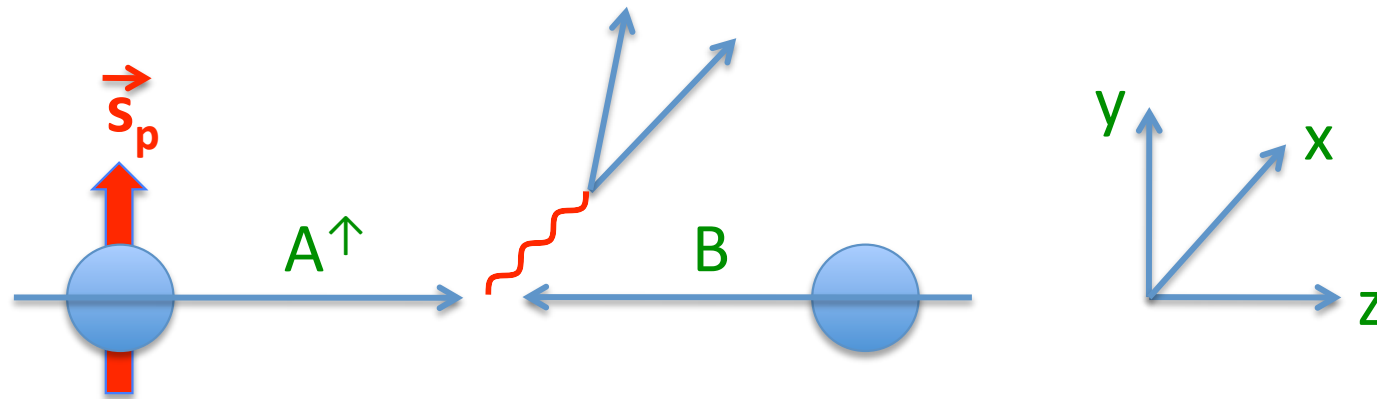
- $\chi^2/\text{d.o.f} = 1.3$ : slightly larger than the usual Gaussian fit. Feel more confident when extrapolated to the whole  $Q$  range



- Only  $u$  and  $d$  quark Sivers functions are constrained by SIDIS data, all the sea quark Sivers functions are not constrained
  - If set all sea quark Sivers functions vanishing, one still obtains the almost the same  $\chi^2/\text{d.o.f}$ .

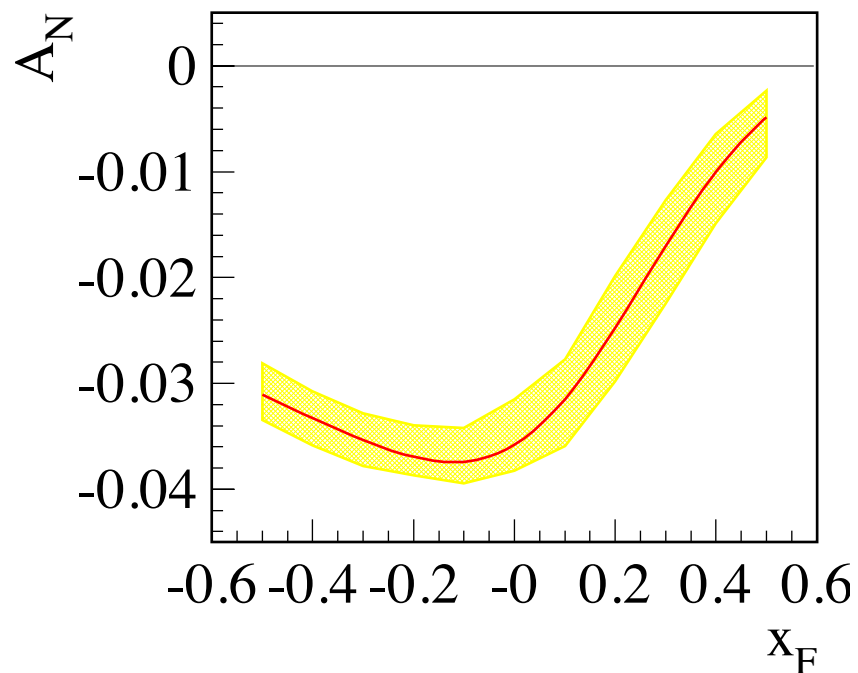
# Some predictions for asymmetries of DY and W

- At 510 GeV RHIC energy (DY:  $p_T$  [0,1],  $Q$  [4,9] W:  $p_T$  [0,3] GeV)

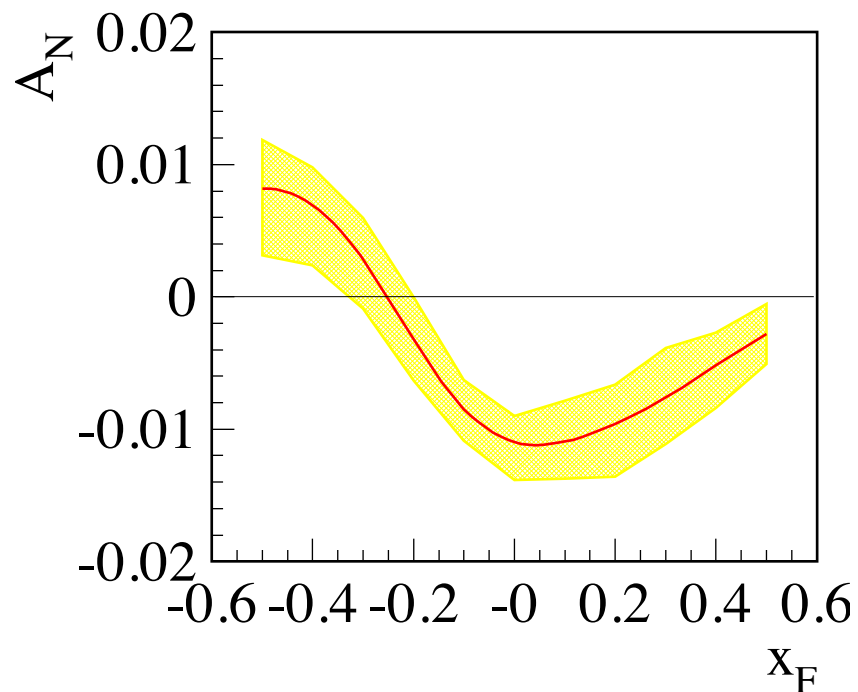


## Predictions for other experiments

- DY at COMPASS: 190 GeV pi- beam



- DY at Fermilab:  $x_F > 0$  polarized beam;  $x_F < 0$  polarized target



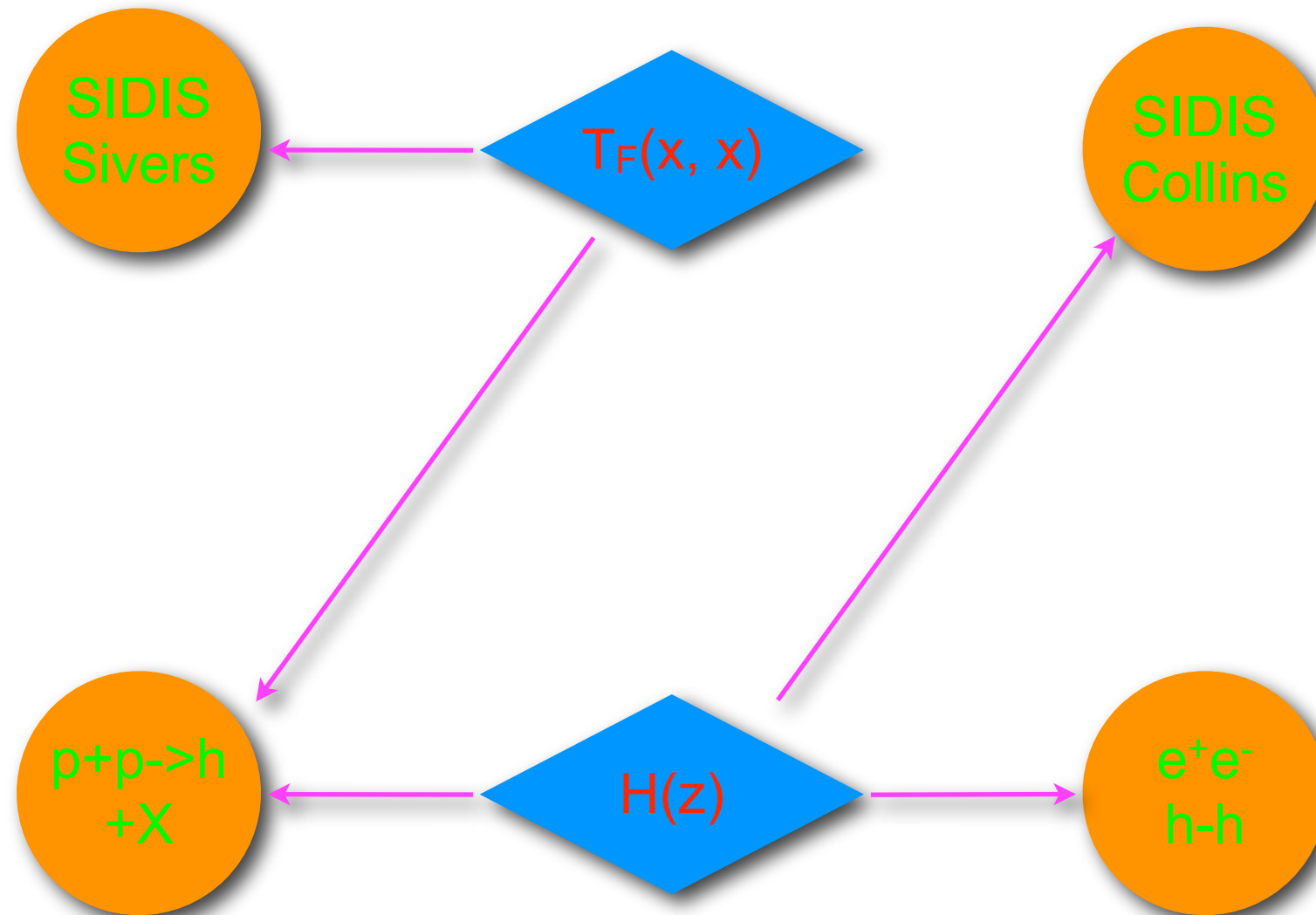


# Roadmap for global analysis of spin asymmetries

Gamberg-Kang-Prokudin, in preparation

TMD evolution

TMD evolution



Collinear evolution

TMD evolution



## Summary

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- Perturbatively, the QCD evolution kernel for TMDs are the same in all existing approaches
- The difficult on the QCD evolution comes from pinning down the non-perturbative part, which has to be fitted from experimental data
- We find some simple non-perturbative form, which can describe all the data on SIDIS, DY, W/Z production
- Use the same non-perturbative form, we extract the Sivers function and predict the asymmetry for DY and W production at RHIC energy



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Thank you